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THE

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THE

COMPLETE MEASURER:

OR, THE

WHOLE ART OF MEASURING.

BEING

A PLAIN AND COMPREHENSIVE TREATISE

ON

PRACTICAL GEOMETRY

AND

MENSURATION.

CONTAINING

The Substance of HAWNEY'S MENSURATION, *newly arranged, adapted to the present improved State of Science, and incorporated with a Variety of original and important Matter.*

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1817.

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PREFACE.

IN the year 1798, the Proprietors of HAWNEY'S Mensuration applied to the Editor to superintend a new edition of that Work, and to make such alterations and additions as would render it useful in schools, without deviating much from the original plan.

The work at that time did not contain any PRACTICAL GEOMETRY, notwithstanding its evident utility; many of the rules were erroneous and defective, and there was only *one* example given to each rule, and that worked at full length.

The paucity of examples excluded the book from our modern schools; the Editor therefore added upwards of *three hundred:* also an extensive chapter of Practical Geometry, and corrected all the rules.

In subsequent editions, several new rules were added, likewise the description and use of the Carpenters Rule, and of the Sliding Rule: the other different articles were, many of them, extended.

This edition * has been entirely re-written, excepting those parts which the Editor had previously introduced into the work; and only seven of the old wood-cuts are retained: the rest, amounting to *one hundred and fifty-three*, are new; so that few traces of the original work are left.

So great a deviation from the *sixteenth* edition, and those which preceded it, will perhaps appear unwarrantable to persons unacquainted with the work; but, if the Editor can secure the approbation of experienced teachers and mathematicians, his time will not have been misemployed, and any apology for the alterations and additions will be unnecessary.

---

* The *twentieth*, and the *fourth*, superintended by the Editor.

It may be proper to state, that not one rule which was contained in the old editions has been omitted, although they are differently expressed; and that every article marked with an asterisk * in the table of contents has been supplied by the Editor, together with all the new figures, and the practical examples, excepting one to each of the old rules, and those at the end of the book.

The Editor has been enabled to extend the Mensuration, by cancelling the arithmetic, which was at the beginning of the old editions, and by contracting the solutions to the examples; a single solution frequently occupied three or four pages.

The Work, thus improved, treats on the mensuration of almost every geometrical figure. Rules might have been given for the mensuration of haystacks, canals, embankments, &c.; but it is presumed that any person who thoroughly understands the different rules in this Treatise, will not be at a loss to measure such figures. In a circular haystack, the lower part may be measured as the frustum of a cone, and the upper part as a cone, or as a parabolic conoid, if the sides do not slope regularly to the top.

The lower part of a long haystack may be measured as a prismoid, and the upper part as a prism, the base or end of which is a triangle, or a parabola. A canal is generally a prism, with a trapezoidal end; or a prismoid, the widest part being uppermost; an embankment is generally the same figure inverted; but it would be an endless task to point out all the different figures that may occur in practice.

No. 1, YORK-BUILDINGS, NEW-ROAD,
ST. MARY-LE-BONE, LONDON, 1817.

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Books written by the Editor of this Work, and sold by the Proprietors.

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LAGRANGE.

Explanation of the Characters made use of in the following Work.

Charact.	Names.	Significations.
$+$	Plus, or more,	the sign of addition, as $2 + 4$ signifies that 2 and 4 are to be added together.
$-$	Minus, or less,	the sign of subtraction, as $8 - 3$, signifies that 3 is to be subtracted from 8.
$\times$	multiplied into or by,	the sign of multiplication, as 7×5, signifies that 7 is to be multiplied into or by 5.
$\div$	divided by,	the sign of division, as $9 \div 3$, signifies that 9 is to be divided by 3; and $\frac{9}{3}$, or 3)9(signifies the same.
$=$	equal to,	the sign of equality, as $9=9$, signifies that 9 is equal to 9; or $\overline{5+4-2}=7$ signifies that 5, increased by 4 and diminished by 2, is equal to 7.
$: \, :: \, :$	Proportion,	$2 : 4 :: 8 : 16$, signifies that 2 is to 4 as 8 is to 16.

m^2, m^3, signifies the square or cube of the letter m.

AC^2, signifies the square of the line AC.

$\sqrt{AC^2 + CD^2}$, signifies the square root of the sum of the squares of AC and CD.

THE

COMPLETE MEASURER.

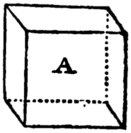
PRACTICAL GEOMETRY.

DEFINITIONS.

1. GEOMETRY is a science which teaches and demonstrates the properties of all kinds of magnitude, or extension; as solids, surfaces, lines, and angles.

2. It is divided into two parts, theoretical and practical. Theoretical Geometry treats of the various properties of extension abstractedly; and practical Geometry applies these considerations to the various purposes of life.

3. *A Solid* is a figure, or body, of three dimensions, viz. length, breadth, and thickness. And its boundaries are superficies, or surfaces. Thus A represents a solid.



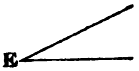
4. *A Superficies*, or surface, is an extension of two dimensions, viz. length and breadth, without thickness: and its boundaries are lines. Thus B represents a surface.

5. *A Line* is a length without breadth, and is formed by the motion of a point. Thus CD represents a line. Hence the extremities of lines are points, as are likewise their intersections.

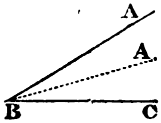
6. *Straight Lines* are such as *cannot* coincide in two points, without coinciding altogether, and a straight line is the shortest distance between two points.

7. *A Point* is the inceptive of magnitude. It is indivisible; and void of length, breadth, or thickness; for were it possessed of either length, breadth, or thickness, it would necessarily become one or other of the figures in definition 3, 4, or 5.

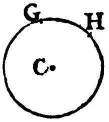
8. *A plane rectilineal Angle* is the inclination, or opening, of two straight lines meeting in a point, as E.



9. One angle is said to be less than another, when the lines which form it are nearer to each other. Take two lines A B and B C touching each other in the point B: conceive these two lines to open, like the legs of a pair of compasses, so as always to remain fixed to each other in B.—While the extremity A moves from the extremity C the greater is the opening or angle A B C; and on the contrary, the nearer you bring them together, the less the opening or angle will be.

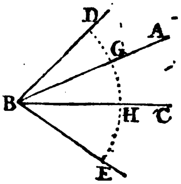


10. *A Circle* is a plane figure contained by one line called the circumference, which is every where equally distant from a point within it, called the centre, as C: and an arc of a circle is any part of its circumference, as G H.



11. The magnitude of an angle does not consist in the length of the lines which form it, but in their opening or inclination to each other. Thus the angle A B C is less than the angle D B E, though the lines A B and C B, which form the former angle, are longer than the lines D B and B E, which form the latter.

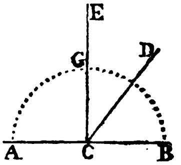
12. When an angle is expressed by three letters, as A B C, the middle letter always stands at the angular point, and the other two letters at the extremities of the lines which form the angle. Thus the angle A B C is formed by the lines A B and C B, that of D B E is formed by the lines D B and B E.



13. Every angle is measured by an arc of a circle, described about the angular point as a centre; thus the arc D E is the measure of the angle D B E, and the arc G H is the measure of the angle A B C.

14. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds. The angles are measured by the number of degrees cut off from the circle, by the lines which form the angles. Thus, if the arc G H contain 20 degrees, or the eighteenth part of the circumference of the circle, the measure of the angle A B C is said to be 20 degrees.

15. When a straight line E C standing upon a straight line A B, makes the adjacent angles A C E and B C E equal to each other; each of these angles is said to be a *right angle*, and the line E C is *perpendicular* to A B. The measure of a right angle is therefore 90 degrees, or quarter of a circle.

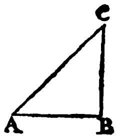


16. *An acute* angle is less than a right angle, as D C B, or E C D.

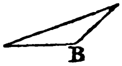
17. *An obtuse* angle is greater than a right angle, as A C D.

18. *A plane Triangle* is a space included by three straight lines, and contains three angles.

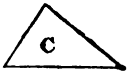
19. *A right angled Triangle* is that which has one right angle in it, as A B C. The side A C, opposite to the right angle, is called the hypothenuse, the side B C is called the perpendicular, and the line A B, on which the triangle stands, is called the base.



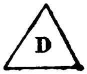
20. *An obtuse angled Triangle* has one obtuse angle in it, as B.



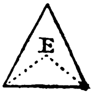
21. *An acute angled Triangle* has all its three angles acute, as C.



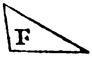
22. *An equilateral triangle* is that which has three equal sides, and three equal angles, as D



23. *An isosceles* triangle has two equal sides, and the third side either greater or less, than each of the equal sides, as E.



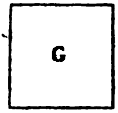
24. *A scalene* triangle has all its three sides unequal, as F.



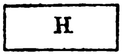
25. *A quadrilateral* figure is a space included by four straight lines, and contains four angles.

26. *A Parallelogram* is a plane figure bounded by four straight lines, whereof those which are opposite are parallel one to the other; that is, if produced ever so far would never meet.

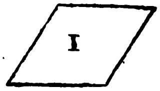
27. *A Square* is an equilateral parallelogram, viz. having all its sides equal, and all its angles right angles, as G.



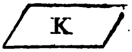
28. *An Oblong*, or rectangle, is a right angled parallelogram, whose length exceeds its breadth, as H.



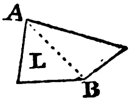
29. *A Rhombus* is a parallelogram having all its sides equal, but its angles are not right angles, as I.



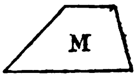
30. *A Rhomboides* is a parallelogram having its opposite sides equal, but its length exceeds its breadth, and its angles are not right angles, as K.



31. *A Trapezium* is a plane figure contained under four straight lines, no two of which are parallel to each other, as L. A line connecting any two angles of the trapezium, as A B, is called a diagonal.

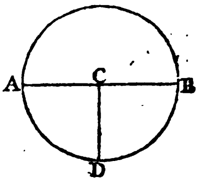


32. *A Trapezoid* is a plane quadrilateral figure having two of its opposite sides parallel, and the remaining two not parallel; as M.

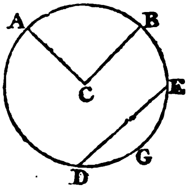


33. *Multilateral* figures, or polygons, have more than four sides, and receive particular names, according to the number of their sides. Thus, a *Pentagon*, is a polygon of five sides; a *Hexagon*, has six sides; a *Heptagon*, seven; an *Octagon*, eight; a *Nonagon*, nine; a *Decagon*, ten; an *Undecagon*, eleven; and a *Duodecagon*, has twelve sides. If all the sides and angles are equal, they are called regular polygons; if unequal, they are called irregular polygons.

34. *The Diameter* of a circle is a straight line drawn through the centre, and terminated by the circumference both ways; thus A B is a diameter of the circle. The diameter divides the surface, and circumference, into two equal parts, each of which is called a *Semicircle*. If a line C D be drawn from the centre C perpendicular to A B, to cut the circumference in D, it will divide the semicircle into two equal parts, A C D and D C B, each of which will be a *Quadrant*, or one-fourth of the circle. The line C D, drawn from the centre to the circumference, is called the *Radius*.



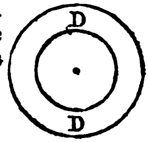
35. A *Sector* of a circle is comprehended under two *Radii*, or semidiameters, which are supposed not to make one continued line, and a part of the circumference. Hence a sector may be either less, or greater, than a semicircle; thus A C B is a sector less than a semicircle, and the remaining part of the circle is a sector greater than a semicircle.



36. *The Chord* of an arc is a straight line, less than the diameter, joining the extremities of the arc; thus D E is the chord of the arc D G E, or of the arc E B A D.

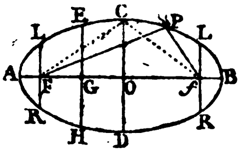
37. *A Segment* of a circle is that part of the circle which is bounded by an arc and its chord, and may be either greater or less than a semicircle.

38. *Concentric circles* are those which have the same centre, and the space included between their circumferences is called a ring, as D.



THE ELLIPSIS.

39. If two pins be fixed at the points F *f*, and a thread P *f* F P be put round them and knotted at P; then if the point P and the thread be moved about the fixed centres F *f*, so as to keep the thread always stretched, the point P will describe the curve PBDACP called *an Ellipsis*.



40. The points or centres F *f*, are called the *Foci*, and their distance from C, or D, is equal to the half of A B.

41. The line A B, drawn through the *Foci* to the curve, is called the *transverse* axis, or the *Axis Major*. The point O, in the middle of the axis A B, is the centre of the ellipsis.

42. The line C D, drawn through the centre O, perpendicular to the transverse axis A B, is called the *conjugate* axis, or the *Axis Minor.*

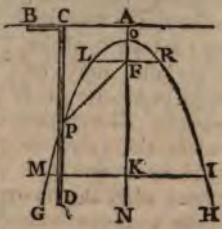
43. The line L R, drawn through the focus F, perpendicular to the transverse axis, is called the *Parameter*, or *Latus Rectum.*

44. A line drawn from any point of the curve, perpendicular to the transverse axis, is called an *Ordinate* to the transverse, as E G. If it go quite through the figure, as E H, it is called a *Double Ordinate.*

45. The extremity of any diameter is called the *Vertex;* thus the vertices of the *transverse* diameter A B, are the points A and B.

46. That part of the diameter between the vertex and the ordinate is called an *Abscissa;* thus G B and A G, are abscisses to the ordinate G E.

THE PARABOLA.



47. If one end of a thread, equal in length to C D, be fixed on the point F, and the other end fixed at D, the end of the square B C D; and the side C B of the square be moved along the straight line A B, so as always to coincide therewith, the string being kept stretched and close to the side of the square P D, the point P will describe a curve H R O L P G, called a *Parabola.*

48. The fixed point F is called the *Focus.*

49. The straight line A B is called the *Directrix.*

50. The straight line O N is the *Axis* of the parabola, and O is the *Vertex.*

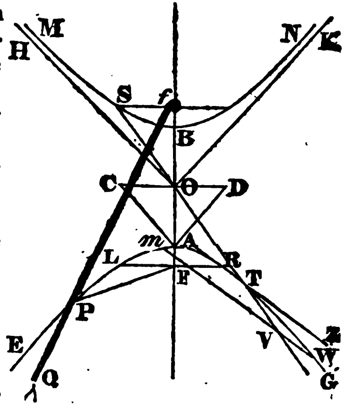
51. A straight line L R, drawn through the focus F, perpendicular to the axis, is called the *Parameter*, or *Latus Rectum.*

52. A straight line I K, drawn from the curve perpendicular to the axis, or parallel to the directrix, is called an *Ordinate;* if it go quite through the figure, as I M, it is called a *double Ordinate.*

53. The part of the axis, K O, between the vertex, O, and ordinate, I K, is called the *Abscissa.*

THE HYPERBOLA.

54. If one end of a long flat ruler f P Q be fastened at the point f on a plane, in such a manner that it may turn freely about the fixed point f as a centre; and one end of a thread F P Q, shorter than the ruler, be fixed at F, and kept constantly stretched along the edge of the ruler, the point P, by the motion of the ruler about f, will describe the curve E P L A, which is one part of an hyperbola. And if the ruler be turned and moved on the other side of the fixed point f, the remaining part A T G of the hyperbola may be described in the same manner. Also, if the end of the ruler be fixed at F and the thread at f, the opposite Hyperbola M S B N may be described.



55. The fixed points f F are called the two *Foci,* and are equi-distant from the centre O of the hyperbola, which is the middle point of A B.

56. The straight line A B passing through the foci, and terminated by the vertices A and B of the hyperbolas, is called the *Transverse Axis,* or *Axis Major,* and is always equal to the difference between f P and F P in every point of the curve.

57. If C D be drawn through the centre O at right angles to A B, and with A as a centre and F O as a distance, two arcs be described cutting C D in the points C and D; then C O D is called the *Conjugate Axis*, or Axis *Minor.*

58. Any straight line S O T drawn through the centre O, and terminated at the opposite sections, as at S and T, is called a *diameter*, and the extremities S and T are its vertices; and if a tangent be drawn through the vertex S, any straight line drawn through the centre O, parallel to this tangent, is the *conjugate* diameter to the preceding one.

69. If any diameter, as S O T, be continued within the curve, that part T V within the curve, is called the *abscissa* to that diameter.

60. A straight line T Z which meets the hyperbola in one point T, but does not cut, it is called a *tangent* to that point.

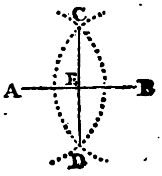
61. Any straight line parallel to a tangent at the vertex T, and terminated by the abscissa, is called an *Ordinate;* and if it cuts the curve in two points, it is called a double ordinate. Thus *m* V is an ordinate to the diameter S T, and M V W is a double ordinate.

62. A straight line, as L F R, drawn through the focus F, at right angles to the Axis Major A B, is called the *Parameter* or *Latus Rectum.*

63. If the vertex A of the hyperbola EPLARTWG be joined to the extremities of the Axis Minor C D, by the straight lines A C, A D, two straight lines drawn through the centre O parallel to A C and A D, as O H and O K will be the *Asymptotes* of the hyperbola M S B N. The *asymptotes* continually approach nearer to the curve, but never meet it.

PROBLEM I.

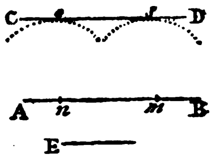
To bisect, or divide, a given straight Line A B *into two equal Parts.*



From the points A and B with any radius, or opening of the compasses, greater than half A B, describe two arcs cutting each other in C and D; draw C D, and it will cut A B in the point E, making A E equal to E B.

PROBLEM II.

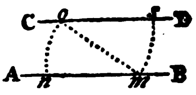
At a given Distance E, *to draw a straight Line* C D *parallel to a given straight Line* A B.



From any two points m n in the line A B with the extent of E in your compasses describe two arcs o s;—draw C D to touch these arcs, without cutting them, and it will be parallel as required.

PROBLEM III.

Through a given Point o, *to draw a straight Line* C D *parallel to a given straight Line* A B.

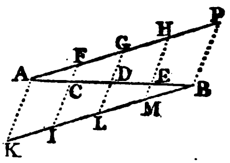


Take any point m in the line A B; with m as a centre and distance m o, describe the arc o n; with n as a centre and the same distance describe the arc s m. Take the arc o n in your compasses, and apply it from m to s; through o, s, draw C D, and it will be parallel as required.

PROBLEM IV.

To divide a given straight Line A B *into any Number of equal Parts.*

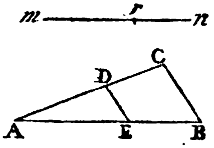
From the ends A and B of the given line, draw two lines A P and B K of any length, parallel to each other. Then set off any number of equal parts from A towards P, and likewise from B towards K; draw lines between the corresponding points HM, GL, FI, &c. and they will divide AB into the equal parts AC, CD, DE, EB.



PROBLEM V.

To divide a given straight Line AB *into two such Parts, as shall be to each other, as* m r *to* r n.

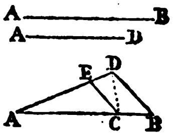
From A draw any line A C equal to *m n*, and upon it transfer the divisions of the line *m n*, viz. *m r* and *r n*. Join B C, and parallel to it draw D E; then A E : E B :: *m r* : *r n*.



PROBLEM VI.

To find a third Proportional to two given Lines, A B, A D.

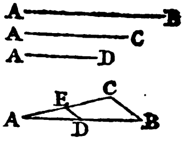
Place the two given lines A B and A D so as to make any angle with each other at A; from A B the greater, cut off a part A C, equal to A D the less; join B D, and draw C E parallel to it, then will A E be the third proportional required, viz. A B : A D :: A D : A E.



PROBLEM VII.

To find a fourth Proportional to three given Lines A B, A C, A D.

Place two of the given lines A B and A C so as to make any angle with each other at A, and join B C. On A B set off the distance A D, and draw D E parallel to B C; then A E is the fourth proportional required, viz.



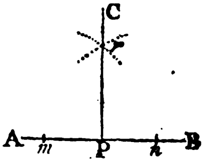
A B : A D :: A C : A E.

PROBLEM VIII.

From a given Point P *in a straight Line* A B *to erect a Perpendicular.*

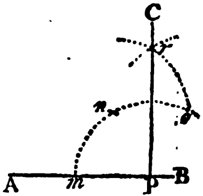
1. *When the Point is in, or near, the Middle of the Line.*

On each side of the point P, take two equal distances P *m*, P *n*; from the points *m* and *n* as centres, with any opening of the compasses greater than P *m*, describe two arcs cutting each other in *r*; through *r*, draw C P, and it will be the perpendicular required.



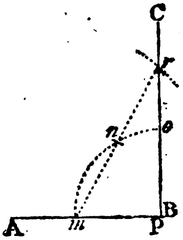
Or thus:

With the centre P, and any opening of the compasses P *m*, describe the arc *m n o*; set off the distance P *m*, from *m* to *n*, and from *n* to *o*; then, from the centres *n* and *o*, with the same, or any other extent of the compasses, describe two arcs intersecting each other in *r*: through *r* draw C P, and it will be the perpendicular required.



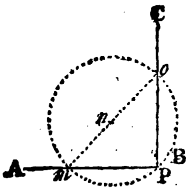
2. *When*

2. *When the Point* P *is at the End of the Line.*



With the centre P and any radius describe the arc *m n o;* set off P *m* from *m* to *n*, and from *n* with the same radius describe an arc *r;* through *m* and *n* draw the line *m n r* to cut the arc in *r;* then through *r* and P draw C P, and it will be the perpendicular required.

Or thus:

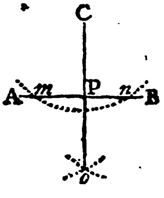


Set one foot of the compasses in P, and extend the other to any point *n*, out of the line A B; from *n* as a centre and distance *n* P, describe a circle cutting A B in *m;* through *m* and *n*, draw *m n o* to cut the circle in *o*, then through *o* draw C P, which will be perpendicular to A B.

PROBLEM IX.

From a given Point C *to let fall a Perpendicular upon a given Line* A B.

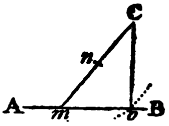
1. *When the Point is nearly opposite to the Middle of the Line.*



From the centre C describe an arc to cut A B in *m* and *n;* with the centres *m* and *n*, and the same radius, describe arcs intersecting in *o;* through C and *o* draw C P, the perpendicular required.

2. *When*

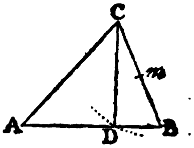
2. *When the Point is nearly opposite the End of the Line.*

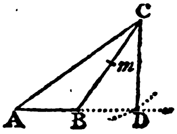


From the given point C draw any line C *m* meeting A B in the point *m*; bisect *m* C in *n*, and with *n* as a centre and radius *n m*, or *n* C, describe an arc cutting A B in *o*, draw C *o*, and it will be the perpendicular required.

PROBLEM X.

In any Triangle A B C *to draw a Perpendicular from any Angle to its opposite Side.*

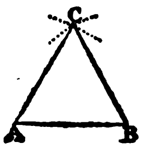




Bisect either of the sides containing the angle from which the perpendicular is to be drawn, as B C in *m*; with the radius *m* B and centre *m* describe an arc cutting A B (produced if necessary) in D, draw C D, and it will be the perpendicular required.

PROBLEM XI.

Upon a given straight Line A B *to describe an equilateral Triangle.*

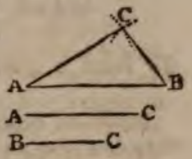


With B as a centre and radius equal to A B describe an arc; with A as a centre and distance A B cross it in C; draw A C and B C, then will A B C be the equilateral triangle required.

PRO-

PROBLEM XII.

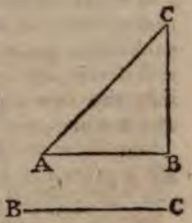
To make a Triangle with three given Lines A B, A C, *and* B C, *of which any two taken together are greater than the third.*



With A as a centre and radius A C describe an arc, with the centre B and distance B C cross it in C, draw A C and B C, then A B C is the triangle required.

PROBLEM XIII.

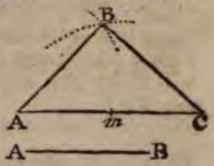
Two Sides A B *and* B C *of a right angled Triangle being given, to find the Hypothenuse.*



From the point B in A B draw B C perpendicular and equal to B C: join A C, and it will be the hypothenuse required.

PROBLEM XIV.

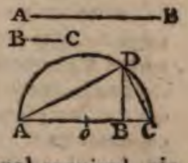
The Hypothenuse A C, *and one Side* A B, *of a right angled Triangle being given, to find the other Side.*



Bisect A C in *m*, with *m* as a centre and distance *m* A describe an arc, with A as a centre and distance A B cross it in B: then A B C is a right angled triangle, and B C is the required side.

PROBLEM XV.

To find a mean Proportional between two given Lines A B *and* B C.



Join A B and B C in one straight line, viz. make A C equal to the sum of them, and bisect it in the point *o*. With the centre *o* and radius *o* A describe a semicircle; at B erect the perpendicular B D, and it will be the mean proportional required, viz.

A B : B D :: B D : B C.

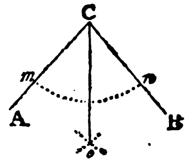
Note. If A D and D C be joined, A D will be a mean proportional between AB and AC, also CD will be a mean proportional between B C and A C, viz.

AB : AD :: AD : AC
and BC : CD :: CD : AC.

PROBLEM XVI.

To bisect a given Angle, viz. to divide it into two equal Parts.

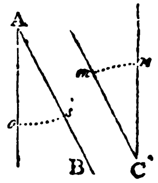
From the centre C with any radius describe an arc *m n*, from *m* and *n*, as centres with the same radius describe two arcs crossing each other in *o*, draw C *o*, and it will divide the given angle A C B into the two equal angles A C *o* and B C *o*.



PROBLEM XVII.

At a given Point A, *in a given Line* A B *to make an Angle equal to a given Angle* C.

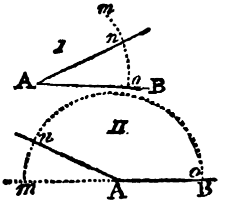
With the centre C and any radius describe an arc *m n*; with the centre A and the same radius describe the arc *o s*: take the distance *m n* in your compasses, and apply it from *s* to *o*; then a line drawn from A through *o* will make the angle A equal to the angle C.



PROBLEM XVIII.

To make an Angle of any proposed Number of Degrees upon a given straight Line, by the Scale of Chords.

Upon the line A B to make an angle of 30 deg. (fig. 1.) Take the extent of 60 degrees from the line of chords, with which and the centre A describe the arc *o m*. Take 30 degrees from the same scale of chords, and set them off from *o* to *n*; through *n* draw A *n*, then *n* A B is the angle required.



To make an angle of 150 degrees, (fig. 2), produce the line B A to *m*, with the centre A and the chord of 60 degrees, describe a semicircle; take the given obtuse angle from 180 degrees, and set off the remainder, viz. 30 degrees from *m* to *n*; through *n* draw A *n*, then *n* A B is the angle required.

PROBLEM XIX.

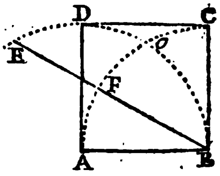
An Angle being given to find how many Degrees it contains, by a Scale of Chords.

With the chord of 60 degrees in your compasses and centre A (fig. 1.) describe the arc *o n*, cutting the two lines which contain the angle in *o* and *n*; take the distance *o n* in your compasses, and setting one foot at the beginning of the chords on your scale, observe how many degrees the other foot reaches to, and that will be the number of degrees contained in the arc *o n*, or angle *n* A B.

If the extent *on* reach beyond the scale, which will always be the case when the angle is obtuse, produce the line B A from A towards *m*, (fig. 2), and measure the arc *m n* in the same manner, the degrees it contains, deducted from 180 degrees, will give the measure of the angle *n* A B.

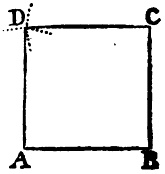
PROBLEM XX.

Upon a given straight Line A B, *to describe a Square.*



With A as a centre and distance A B describe the arc E B, with B as a centre and the same extent describe the arc A C cutting the former in *o*; make *o* E equal to B *o*, and draw B E; make *o* C and *o* D each equal to A F, or F *o*, and join the points A D, D C and C B, then will A B C D be the square required.

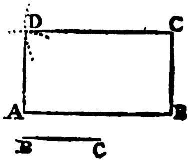
Or thus:



Draw B C perpendicular, and equal, to A B; with the extent A B and one foot in A describe an arc; with the same extent and one foot in C cross it in D, join A D and D C, then A B C D is the square required.

PROBLEM XXI.

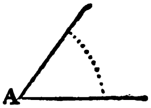
To make an Oblong, or rectangled Parallelogram, of a given Length A B, *and Breadth* B C.



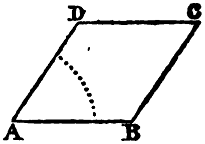
Draw BC perpendicular to AB; with the centre A and distance B C describe an arc, with the centre C and distance A B cross it in D; join A D and D C, then A B C D is the oblong required.

PROBLEM XXII.

***Upon a given straight Line* A B *to describe a Rhombus, having an Angle equal to the given Angle* A.**



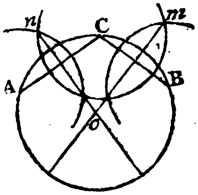
Upon A B make the angle D A B equal to the given angle A; also make A D equal to A B. Then with D and B as centres and radius A B, describe arcs crossing each other in C; join D C and B C, then A B C D is the rhombus required.



PRO-

PROBLEM XXIII.

To find the Centre of a given Circle.

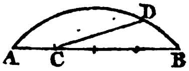


Take any three points A, C, B in the circumference of the circle, and join AC and BC. Bisect AC and BC with the perpendiculars *n o*, and *m o*, meeting each other in the point *o*. Then *o* is the centre required.

Note. By this Problem it will be easy to describe a circle through any three given points that are not in the same straight line: or, to describe a circle about a given triangle. Also, if any arc of a circle be given, the whole circle may be readily described.

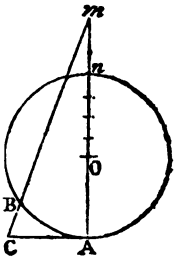
PROBLEM XXIV.

To draw a straight Line equal to any given Arc, of a Circle, A B.



Divide the chord A B into four equal parts; set one part A C on the arc from B to D: draw C D, and it will be *nearly* equal to half the length of the arc A D B.

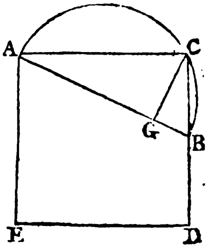
Or thus:



Through the point A, and *o* the centre of the circle draw A *m*; divide *o n* into four equal parts, and set off three of them from *m* to *n*. Draw A C perpendicular to A *m*, then through *m* and B draw *m* C; then will A C be equal to the length of the arc A B *very nearly*.

PROBLEM XXV.

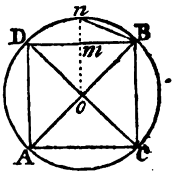
To make a Square equal in Area to a given Circle.



Divide the diameter A B into fourteen equal parts, and make A G equal to eleven of these parts; erect the perpendicular G C, and join A C; then the square A E D C, formed upon A C, is equal to the whole circle whose diameter is A B, *exceedingly near the truth.*

PROBLEM XXVI.

In a given Circle to describe a Square.

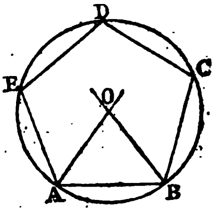


Draw any two diameters A B and C D perpendicular to each other, then connect their extremities, and that will give the inscribed square A D B C.

Note. If a side of the square, as D B, be bisected in *m*, and a line *o m n* be drawn from the centre of the circle to cut the circumference; then if the line B *n* be drawn, it will be the side of an octagon inscribed in the circle.

PROBLEM XXVII.

To make a regular Polygon on a given Line A B.



Divide 360 degrees by the number of sides contained in your polygon, subtract the quotient from 180 degrees, and the remainder will be the number of degrees in each angle of the polygon. From each end of A B draw lines A O, B O, making angles with the given line equal to half the angle of the polygon:

then with the centre O and radius O A describe a circle, to the circumference of which apply continually the given side A B.

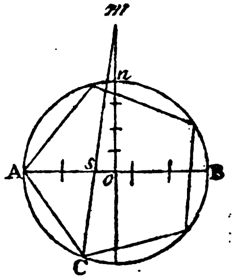
Or thus:

Take the given line A B from any scale of equal parts; multiply the side of your polygon by the number in the third column of the following Table, answering to the given number of sides, and the product will give you the length of A O or O B, with which proceed as above.

No. of Sides.	Name of the Polygon.	Rad. of the Circumscribing Circle.	Angle O A B, or, O B A.	
3	Trigon	·5773503	30	
4	Tetragon	·7071068	45	
5	Pentagon	·8506508	54	
6	Hexagon	1, *Side=Radius.*	60	Degrees.
7	Heptagon	1·1523825	$64\frac{2}{7}$	
8	Octagon	1·3065630	$67\frac{1}{2}$	
9	Nonagon	1·4619022	70	
10	Decagon	1·6186340	72	
11	Undecagon	1·7747329	$73\frac{7}{11}$	
12	Duodecagon	1·9318516	75	

PROBLEM XXVIII.

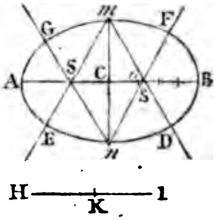
In a given Circle to inscribe any regular Polygon; or, to divide the Circumference of a given Circle into any Number of equal Parts.



Divide the diameter A B into as many equal parts as the figure has sides; from the centre *o* draw the perpendicular *o m*, divide the radius *o n* into four equal parts, and set off three of these parts from *n* to *m*; from *m*, through the second division *s*, of the diameter A B, draw *m* C; join A C, and it will be the side of the polygon required.

PROBLEM XXIX.

Given the two Diameters of an OVAL, *a Figure resembling the Conic Section called an* ELLIPSIS, *to describe it.*



Bisect the longer diameter A B in the point C by the line *m n*, bisect the shorter diameter H I in K, set off H K or I K from C towards *m* and *n*.

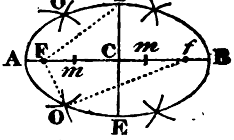
Make A *o* equal to H I, divide the remaining part *o* B into three equal parts, and set off two of them from C to S.

With S as a centre, and radius S S, cross the line *m n*, in *m* and *n*, through S draw *m* S E, *m* S D, *n* S G, and *n* S F, of an indefinite length.

With the radius A S and centre S describe the arcs E A G and F B D; also with *n* as a centre and radius *n* G describe the arc G F, passing through the extremity of the shorter diameter, and meeting B F in the point F; in a similar manner, with *m* as a centre, and radius *m* E or *m* D, describe the arc E D, which will complete the *oval* required.

PROBLEM XXX.

The transverse and conjugate Diameters of an ELLIPSIS *being given, to find the Foci and to construct the Ellipsis.*



Draw the transverse and conjugate diameters A B and D E, bisecting each other at right angles in the centre C.

With the radius A C and centre D describe an arc cutting A B in F, *f*, these two points will be the *foci* of the ellipsis.

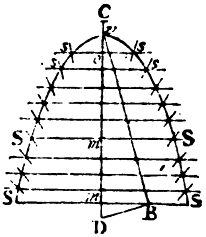
Take any number of points *m*, *m*, &c. in the transverse diameter A B; then, with the radii A *m*, *m* B, and centres F, *f*, describe arcs intersecting each other in O, O, &c. these points of intersection will all be situated in the curve, which may be drawn through them with a steady hand.

Or,

Having found the two foci, the ellipsis may be described by the 39th definition; for, any two straight lines F O, *f* O, drawn from the two foci F, *f*, to the curve at O, are together equal to the transverse diameter A B of the ellipsis.

PROBLEM XXXI.

Given the Abscissa v m, and ordinate s m, of a Parabola, to construct it.



Bisect the given ordinate s *m* in B; join B *v* and draw B D perpendicular to it, meeting the abscissa produced in D.

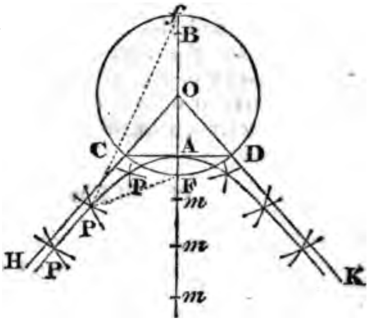
Make *o v* and *v* C, each equal to D *m*, then will *o* be the *focus* of the parabola.

Take any number of points *m m*, &c. in the abscissa, through which draw the double ordinates s *m* s, &c. of an indefinite length. With the radii *o* C, *m* C, &c. and centre *o*, describe arcs cutting the corresponding ordinates in the points s s, &c. and the curve s *v* s, drawn through all the points, will be the parabola required.

PRO-

PROBLEM XXXII.

Given the transverse and conjugate Diameters of an Hyperbola, to describe the Curve and its Asymptotes.



Let C D be the conjugate diameter, bisect it in A and draw A B at right angles to C D, then A B will be the transverse diameter. These two lines may be of any given length.

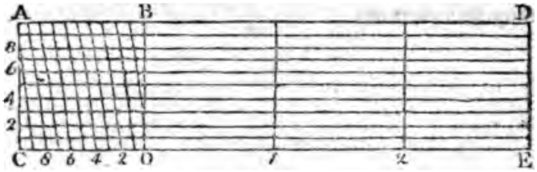
Bisect A B in O, and with the radius O C or O D and centre O describe the circle C F D *f*, cutting A B produced in *f* and F. These two points will be the two foci of the hyperbola.

In B A produced take any number of points *m*, *m*, &c. and from *f* and F as centres with the distances B *m*, and A *m* as radii describe arcs cutting each other in P, P, &c. these points of intersection will all be situated in the curve, which may be drawn through them with a steady hand.

If straight lines O H, O K, be drawn from the point O, the middle of the transverse diameter, through C and D, the extremities of the conjugate, they will be the *asymptotes* of the hyperbola. See Definition 63.

PROBLEM XXXIII.

To construct a Diagonal Scale.



Draw eleven lines of equal length and at equal distances from each other, as in the above figure; divide the two outer lines, as A D, C E, into any convenient number of equal parts, according to the largeness you intend your scale. Join these parts by straight lines, as A C, B *o*, &c. first taking care that the corners C, A, D, E, are all square. Again divide the lengths A B and C *o* into 10 equal parts, join these parts by diagonal lines, viz. from the point A to the first division on C *o*, and from the first division in A B, to the second division in C *o*, &c. as in the figure, and number the several divisions.

The chief use of such a scale as this, is to lay down any line from a given measure; or to measure any line, and thereby to compare it with others. If the large divisions in *o* E be called units, the small divisions in C *o* will be 10ths, and the divisions in the altitude *o* B will be 100th parts of an unit. If the large divisions be tens, the others will be units, and tenth parts. If the large divisions be hundreds, the others will be tens and units, &c. each set of divisions being tenth parts of the former ones.

For example, suppose it were required to take off 244 from the scale: fix one foot of the compasses at 2 of the larger divisions in *o* E, and extend the other to the number 4 in C *o*; then move both points of the compasses by a parallel motion, till you come to the fourth long line, taking care to keep the right hand point in the line marked 2; then open the compasses a small matter, till the left hand foot reaches to the intersection of the two lines marked 4, 4, and you have the extent of the number required. In a similar manner, any other number may be taken off.

THE
COMPLETE MEASURER.

MENSURATION OF SUPERFICIES.

THE area of any plane figure, is the measure of its surface, or of the space contained within the boundaries of that surface, without any regard to thickness, and is estimated by the number of squares contained therein; the side of those squares being either an inch, a foot, a yard, a link, a chain, &c.

And hence the area is said to be so many square inches, square feet, square yards, square links, or square chains, &c.

Our common measures of length are given in the first table below, and the second table of square measure is taken from it, by squaring the several numbers.

I.

Lineal Measure.

12	Inches - -	1 Foot.
3	Feet - -	1 Yard.
6	Feet - -	1 Fathom.
16½ 5½	Feet Yards	1 Pole or Rod.
40	Poles - -	1 Furlong.
8	Furlongs -	1 Mile.

II.

Square Measure.

144	Inches -	1 Foot.
9	Feet - -	1 Yard.
36	Feet - -	1 Fathom.
272¼ 30¼	Feet Yards	1 Pole or Rod.
1600	Poles - -	1 Furlong.
64	Furlongs -	1 Mile.

Land is measured by a chain, called *Gunter's Chain*, of 4 poles, or 22 yards in length, and consists of 100 equal links, each link being $\frac{22}{100}$ of a yard in length, or 7·92 inches. Ten square chains, or 10 chains in length and one in breadth, make an acre; or 4840 square yards, 160 square poles, or 100,000 square links, each being the same in quantity. Forty perches, or square poles, make a rood, and 4 roods make an acre.

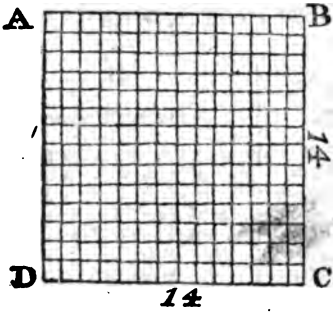
The length of lines measured with a chain, are *generally* set down in links, as whole numbers; every chain being 100 links in length. Therefore after the dimensions are squared, or the superficies is found, it will be in square links; when this is the case, it will be necessary to cut off five of the figures on the right hand for decimals, and the rest will be acres. These decimals must be then multiplied by 4 for roods, and the decimals of these again, after five figures are cut off, by 40 for perches.

PROBLEM I.

To find the Area of a Square.

RULE. Multiply the side by itself, and the product will be the area.

Example 1. Let A B C D be a square, each side being 14. Required the area.



14
14
56
14
196 the area.

2. What is the area of a square whose side is 35·25 chains? *Ans.* 124 acres 1 rood 1 perch.

3. Required the area of a square whose side is 5 feet 9 inches. *Ans.* 33 feet 0 inches 9 parts.

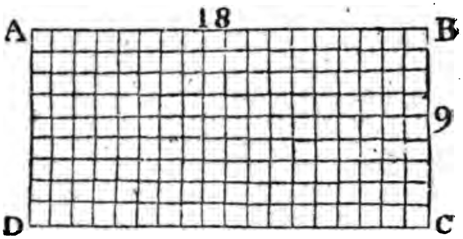
4. What is the area of a square whose side is 3725 links? *Ans.* 138 acres 3 roods 1 perch.

PROBLEM II.

To find the Area of a Rectangle.

RULE. Multiply the length by the breadth, and the product is the area.

Example 1. Let ABCD be a rectangle, the length whereof is 18 feet, and the breadth 9 feet. Required the area. *Ans.* 18 × 9 = 162 feet the area.



2. What is the superficial content of a rectangular board, whose length is 14 feet 6 inches, and breadth 4 feet 9 inches. *Ans.* 68 feet 10 inches 6 parts.

3. Required the area of a rectangular piece of ground, whose length is 1375 links, and breadth 950 links. *Ans.* 13 acres 0 roods 10 perches.

4. What is the superficial content of a rectangle which is 2 feet 10 inches 6 parts long, and 9 inches broad? *Ans.* 2 feet 1 inch 10 parts 6 seconds.

PROBLEM III.

To find the Area of a Rhombus, or of a Rhomboides.

RULE. Multiply the length by the perpendicular breadth, and the product will be the area *.

Perpend.

---

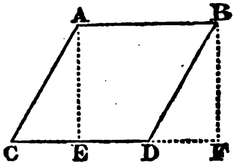
* *Demonstration.* Because parallelograms upon the same base and between the same parallels are equal to each other, (EUCLID 35. of

Example 1. Let ABCD be a rhombus whose sides are each 15·5 feet, and the perpendicular EA = 13·42. Required the area.

Perpend. AE = 13·42
The length CD = 15·5

6710
6710
1342

Area = 208·018



2. What is the area of a rhombus, whose length is 6·2 chains, and perpendicular height 5·45 chains?
Ans. 3 acres 1 rood 20·64 perches.

3. The length of a rhombus is 12 feet 6 inches, and perpendicular height 9 feet 7 inches, what is the area? *Ans.* 119 feet 9 inches 6 parts.

4. The length of a rhombus is 725 links, and perpendicular height 635 links, what is the area?
Ans. 4 acres 2 roods 16·6 perches.

5. A piece of ground, in the form of a rhomboides, measures 4784 links in length, and its perpendicular breadth is 1908 links, what is its area?
Ans. 91 acres 1 rood 4·5952 perches.

6. Required the area of a rhomboides, whose length is 12 feet 6 inches, and perpendicular breadth 5 feet 6 inches. *Ans.* 68 feet 9 inches.

7. How many square yards of painting are in a rhomboides, whose length is 37 feet, and breadth 5 feet 3 inches? *Ans.* $21\frac{7}{12}$ yards.

---

35. of I.) the parallelogram ABDC is equal to the rectangle ABFE.

But the area of the rectangle ABFE is equal to AE × EF (*Prob.* II), that is, = AE × CD which is the rule.

Note. If any two sides of a parallelogram be multiplied together, and that product be multiplied by the natural sine of their contained angle, the last product will give the area of the parallelogram, viz. AC × CD × natural sine of the angle C = the area.

PRO

PROBLEM IV.

To find the Area of a Triangle.

Rule. Multiply the base by the perpendicular height, and half the product will be the area *.

Example 1. If the base of a triangle be 15·4 inches, and the perpendicular 7·8 inches, what is the area?

Ans. (15·4 × 7·8) ÷ 2 = 60·06 inches.

2. The base of a triangle is 28·2 yards, and the perpendicular height 18·4 yards, what is the area?

Ans. 259·44 square yards.

3. There is a triangular field whose base measures 1236 links, and perpendicular 731 links; how many acres does it contain?

Ans. 4 acres 2 roods 2·8128 perches.

4. What is the area of a triangle whose base is 18 feet 4 inches, and perpendicular height 11 feet 10 inches? *Ans.* 108 feet 5 inches 8 parts.

PROBLEM V.

To find the Area of a Triangle by having the three Sides given.

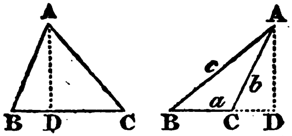
Rule. Add the three sides together, and take half that sum; subtract each side separately from the half sum, then multiply the half sum and the three remainders together, and the square root of the last product will give the area †.

---

* *Demonstration.* The truth of this rule is obvious, because a triangle is the half of a parallelogram of the same base and altitude. (Euclid, 41. of I.)

† *Demonstration.* $AC^2 = (AB^2 + BC^2) - (2BC \times BD)$ Euclid 13 of II. $\therefore BD = \frac{AB^2 + BC^2 - AC^2}{2BC}$, but $AB^2 - BD^2 = AD^2$ Euclid 47 of I. viz. $\sqrt{AB^2 - \left(\frac{AB^2 + BC^2 - AC^2}{2BC}\right)^2} = AD$.

Example 1. Let ABC be a triangle, whereof the sides AB = 43·3, AC = 20·5, and BC = 31·2. Required the area.



AB = 43·3	4·2	*first* remainder.
AC = 20·5	27·0	*second* remainder.
BC = 31·2	16·3	*third* remainder.

Sum - 95·0

Half Sum 47·5

Then $\sqrt{47{\cdot}5 \times 4{\cdot}2 \times 27 \times 16{\cdot}3} = \sqrt{87799{\cdot}95}$ = 296·31 the area.

= AD. Now $\frac{AD \times BC}{2}$ = the area of the triangle by Problem IV, $= \sqrt{AB^2 - \overline{\frac{AB^2 + BC^2 - AC^2}{2BC}}^{\,2}} \times \frac{1}{2}BC$.

Let AB = c, BC = a, and AC = b, then the area of the triangle $= \sqrt{c^2 - \overline{\frac{c^2 + a^2 - b^2}{2a}}^{\,2}} \times \frac{1}{2}a = \frac{1}{4}$
$\sqrt{2c^2a^2 - c^4 + 2c^2b^2 - a^4 + 2a^2b^2 - b^4} = \frac{1}{4}$
$\sqrt{(a^2 + 2ab + b^2 - c^2) \times (-a^2 + 2ab - b^2 + c^2)}$
$= \frac{1}{4}\sqrt{(\overline{a+b}^2 - c^2) \times (c^2 - \overline{a-b}^2)} = \frac{1}{4}$
$\sqrt{\overline{a+b+c} \times \overline{a+b-c}; \times \overline{a+c-b} \times \overline{b+c-a};} =$
$\sqrt{\frac{a+b+c}{2} \times \frac{a+b+c}{2} - c \times \frac{a+b+c}{2} - b \times \frac{a+b+c}{2} - a}$
which is the rule.

Cor If the triangle be equilateral, the area becomes $\frac{1}{4}a^2\sqrt{3}$, the side being represented by a.

Note. If any two sides of a triangle be multiplied together, and that product by the natural sine of the contained angle, the last product will be double the area of the triangle. See the note Prob. III.

2. What is the area of a triangle whose sides are 50, 40, and 30? *Ans.* 600.

3. The sides of a triangular garden are 41, 29, and 56 yards, what is the area thereof?

Ans. 574·34 yards.

4. How many acres are contained in a triangle whose three sides are 4900, 5025, and 2569 links?

Ans. 61 acres 1 rood 39·68 perches.

5. A field of a triangular form, whose sides are 380, 420, and 765 yards, lets for 55 shillings per acre; what is the annual rent?

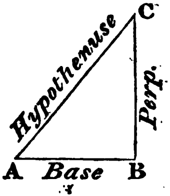
Ans. The area is 44699·0347 yards, or 9·235337 acres, which at 55 shillings per acre, amounts to £25 : 7 : 11¼·28, the annual rent.

PROBLEM VI.

Any two Sides of a right angled Triangle being given, to find the third Side.

Rule 1. To the square of the base add the square of the perpendicular, the square root of the sum will give the hypothenuse, or longest side.

2. Multiply the sum of the hypothenuse and one side, by their difference; the square root of the product will give the other side *.



Example 1. Given the base A B 14·1, the perpendicular B C 12, what is the length of the hypothenuse A C?

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* The truth of this rule is evident from Euclid 47 of I.

14·1 the base A B.
14·1
———
141
564
141
———
198·81 square of A B.
144· square of B C.
———
342·81 sum of the squares, the square root of which extracted gives A C 18·51513 +.

12 the perpend. B C.
12
———
144 square of B C.

Example 2. Given the base A B 14·1, the hypothenuse A C 18·51513 required the perpendicular B C.

18·51513 the hypoth. A C.
14·1 the base A B.
———
32·61513 sum
4·41513 difference.
———
144·0000389169 product of the sum and difference, the square root of which is 12, the perpendicular B C.

3. The base of a right-angled triangle is 24, and the perpendicular 18, what is the hypothenuse?
Ans. 30.

4. The wall of a fort standing on the brink of a river is 42·426 feet high, the breadth of the river is 23 yards; what length must a cord be to reach from the top of the fort across the river? *Ans.* 27 yards.

5. The hypothenuse of a right-angled triangle is 30, and the perpendicular 18, what is the base?
Ans. 24.

6. A ladder, 50 feet long, will reach to a window 30 feet from the ground on one side of a street; and without moving the foot will reach a window 40 feet high on the other side. The breadth of the street is required. *Ans.* 23⅓ yards.

7. A line of 380 feet will reach from the top of a precipice, which stands close by the side of a brook,

to the opposite bank, and the precipice is 128 feet high? how broad is the brook? *Ans.* 357·79 feet.

8. If a ladder, 50 feet in length, exactly reach the coping of a house, when the foot is 10 feet from the upright of the building; how long must a ladder be to reach the bottom of the second floor window, which is 17·9897 feet from the coping, the foot of this ladder standing 6 feet from the upright of the building; and what is the height of the wall of the house?

Ans. The height of the wall is 48·9897 feet, and the length of a ladder to reach the second floor window must be 31·5753 feet.

PROBLEM VII.

To find the Area of a Trapezium.

Rule*. Divide the trapezium into two triangles by a diagonal drawn from two of its opposite angles; then find the areas of these triangles, and add them together.

Or,

If two perpendiculars be drawn from the opposite angles to the diagonal, multiply the sum of these perpendiculars by the diagonal, and half the product will give the area.

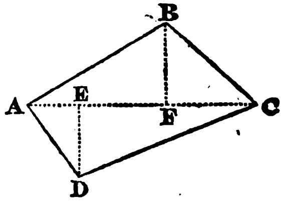
---

* *Demonstration.* The area of the triangle ABC $= \frac{AC \times BF}{2}$, and the area of the triangle ADC $= \frac{AC \times DE}{2}$. The sum of these areas is equal to the area of the whole trapezium, viz. $\frac{AC \times BF}{2} + \frac{AC \times DE}{2} = \frac{BF + DE}{2} \times AC$, which is the second part of the rule.

Note. If the trapezium can be inscribed in a circle, the sum of its opposite angles will be equal to two right angles, (Euclid 22 of III.) and its area may be found by the following rule, viz. Add the four sides together, and take half the sum, from which half sum subtract each side separately. The square root of the product of the four remainders will give the area of the trapezium.

Example 1. Let A B C D be a trapezium, the diagonal A C of which is 80·5, and the perpendiculars B F = 30·1, and D E = 24·5. Required the area.

B F = 30·1
D E = 24·5
Sum 54·6
Half 27·3
A C = 80·5
1365
21840
Area 2197·65



2. There is a field in the form of a trapezium, whose diagonal is 1660 links, the perpendiculars 702 and 712 links, what is the area?

Ans. 11 acres 2 roods 37·792 perches.

3. What is the area of a trapezium whose diagonal is 34 feet 9 inches, and the two perpendiculars 19 feet 9 inches and 8 feet 9 inches?

Ans. 495 feet 2 inches 3 parts.

4. Required the area of a four-sided field, whose south side is 2740 links, east side 3575 links, north side 3755 links, west side 4105 links, and the diagonal from south-west to north-east 4835 links.

Ans. 123 acres 0 rood 11·8672 perches.

5. Suppose in the trapezium A B C D (see the foregoing figure) the side A B to be 15, B C 13, C D 14, and A D 12; also the diagonal A C 16; what is the area thereof? *Ans.* 172·5247.

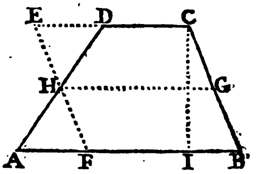
6. Suppose in the trapezium A B C D, on account of obstacles, I could only measure as follow, viz. the diagonal A C 378 yards, the side A D 220 yards, and the side B C 265 yards: but it is known that the perpendicular D E will fall 100 yards from A; and the perpendicular B F will fall 70 yards from C; required the area in acres.

Ans. The perpendicular DE will be 195·959 yards, BF 255·5875 yards; and the area of the trapezium 85342·2885 yards, or 17·6327 acres; or 17 acres 2 roods 21 perches.

PROBLEM VIII.

To find the Area of a Trapezoid.

RULE *. Multiply half the sum of the two parallel sides by the perpendicular distance between them, and the product will be the area.



Example 1. Let ABCD be a trapezoid, the side AB = 23, DC = 9·5, and CI = 13, what is the area?

First, the sum of AB = 23, and DC = 9·5, is 32·5; the half whereof is 16·25, which multiplied by CI = 13, gives the product 211·25 for the area required.

2. Required the area of a trapezoid whose parallel sides are 750 and 1225 links, and the perpendicular distance between them 1540 links?

Ans. 15 acres 0 roods 33·2 perches.

3. What is the area of a trapezoid whose parallel sides are 12 feet 6 inches, and 18 feet 4 inches; and

---

* *Demonstration.* Bisect AD in H, through H draw FE parallel to BC, and produce CD to E; also draw GH parallel to AB or CD.

Now AH is equal to HD, (by construction) and the angle AHF is equal to the angle EHD (EUCLID 15 of I), also the angle HED is equal to the angle HFA (EUCLID 29 of I). Therefore the triangles being equiangular, and having one side equal, are equal in all respects (EUCLID 26 of I); consequently ED is equal to AF.

Again, HG is half the sum of EC and FB, for it is equal to each of them, consequently it is half the sum of AB and DC, but FB multiplied by CI is the area of the figure ECBF, or of its equal ADCB, consequently HG multiplied into CI is the area thereof; which was to be proved.

the perpendicular distance between them 7 feet 9 inches? *Ans.* 119 feet 5 inches 9 parts.

4. A field in the form of a trapezoid, whose parallel sides are 6340 and 4380 yards, and the perpendicular distance between them 121 yards, lets for £83 : 1 : $7\frac{1}{2}$ per annum, what is that per acre? *Ans.* £1 : 11.

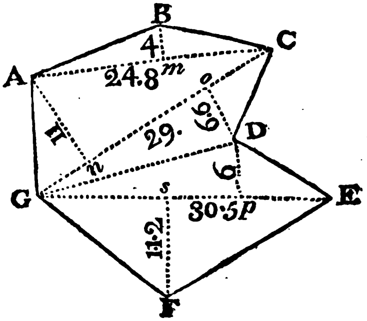
PROBLEM IX.

To find the Area of an irregular Polygon.

Rule. Irregular figures, or polygons, are such as have more sides than four, and the sides and angles unequal. To find the area, draw diagonals dividing the figure into trapeziums, or trapezoids and triangles, and find the areas of each of these separately. Add these areas together, and the sum will be the area of the polygon.

Example. Required the area of the irregular figure A B C D E F G A, the following lines being given.

A C = 24·8
B *m* = 4
G C = 29
A *n* = 11
D *o* = 6·6
G E = 30·5
D *p* = 6
F *s* = 11·2



24·8 = base A C.
2 = half perpend. B *m*

49·6 = area of A B C.

11·2 = perpend. F *s*
6· = D *p*

17·2 sum.

8·6 half sum.

11 = perpend. A *n*
6·6 = D *o*

17·6 sum

8·8 half
29 = diagonal C G.

792
176

255·2 = area of A C D G.

30·5 = diagonal G E.
8·6

1830
2440

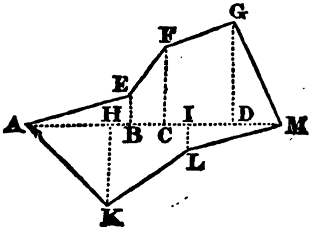
262·30 = area of G D E F.
255·2 = area of A C D G.
49·6 = area of A B C.

Ans. 567·10 sum of the areas.

2. In a five-sided field A B C D E A there is given A B = 2735 links, B C = 3115, C D = 2370, D E = 2925, and E A = 2220; also the diagonal from C to A = 3800, and from C to E = 4010 links. Required the area.

Ans. 117 acres 2 roods 28 perches.

3. In the figure annexed, there are given in links,



AB = 440, BE = 70 | AH = 315, HK = 350
BC = 145, CF = 320 | HI = 295, IL = 50
CD = 405, DG = 470 | IM = 410,
DM = 30,

Required the area. *Ans.* 3 acres 1 rood 16 perches.

PROBLEM X.

To find the Area of a regular Polygon.

*Rule I. Add all the sides together, and multiply the sum by a perpendicular drawn from the centre of the polygon to the middle of one of the sides, and half the product will be the area.

† Rule II. Multiply the square of the side of the polygon, by the number standing opposite to its name in the following table, under the word area, and the product will give the area of the polygon.

---

* Every regular polygon is composed of as many equal triangles as it has sides; consequently if the area of one of these triangles be multiplied by the number of sides in the polygon, the product will give the area of the whole figure. But the area of each of the triangles is equal to the product of the perpendicular and half the base; therefore the product of the perpendicular and half the sum of the sides of the polygon, will give the area.

† All regular polygons, of the same number of sides, are similar to each other, and similar figures are to each other as the squares of their like sides (Euclid 20 *of* VI) ∴ 1^2 : the area in the table :: the square of the side of the polygon : the area.

The table is calculated by trigonometry, thus for the pentagon, divide 360 by 5, the number of sides, and the quotient is 72 degrees for the angle AFB, the half of which is 36, the angle AFG. Then

Sine AFG = 36° - - - - - - 9·7692187
: AG = ·5 half the side - - - - —1·6989700
:: Sine FAG = 54° - - - - - 9·9079576
: The perpendicular FG = ·688191 - - —1·8377089

Then (·688191 × 5) ÷ 2 = 1·720477 the area of a pentagon whose side is 1. In a similar manner all the other numbers are calculated.

Or,

Multiply the side of the polygon, by the number standing opposite to its name in the following table, under the word perpendicular, and the product will give the perpendicular from the centre of the polygon to the middle of one of its sides. Then proceed by the first rule to find the area.

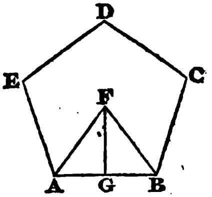
No. of Sides.	Names.	I. Areas. The Side 1.	II. Perpend. The Side 1.
3	Trigon	·433013	·2886751
4	Tetragon	1·000000	·5000000
5	Pentagon	1·720477	·6881910
6	Hexagon	2·598076	·8660254
7	Heptagon	3·633912	1·0382617
8	Octagon	4·828427	1·2071068
9	Nonagon	6·181824	1·3737387
10	Decagon	7·694209	1·5388418
11	Undecagon	9·365640	1·7028437
12	Duodecagon	11·196152	1·8660254

Example 1. The side of a regular pentagon is 25 yards, and the perpendicular from the centre to the middle of one of the sides is 17·204775, required the area.

By Rule I. $\overline{25 \times 5} \times 17{\cdot}204775 = 1075{\cdot}2984375$, *Ans.*

By Rule II. $25^2 \times 1{\cdot}720477 = 1075{\cdot}298125$ yards, *Ans.*

Or, $25 \times {\cdot}6881910 = 17{\cdot}204775$ the perpendicular FG, hence the area is found as above by Rule I.



2. The side of a hexagon is 14·6, and the perpendicular from the centre to the middle of one of the sides = 12·64397, required the area.

Ans. 553·805886.

3. Required the area of an octagon whose side is 9·941, and perpendicular 12. *Ans.* 477·168.

4. Required the area of a heptagon, whose side is 19·38, and perpendicular 20·1215. *Ans.* 1364·841345.

5. Required the area of a decagon, whose side is 20. *Ans.* 3077·6836.

6. Required the area of a duodecagon, whose side is 102, and perpendicular 190·3345908. *Ans.* 116484·7695696.

7. Required the area of a nonagon, whose side is 40, and perpendicular 54·949548. *Ans.* 9890·91864.

8. Required the area of an undecagon, whose side is 20, and perpendicular 34·056874. *Ans.* 3746·25614.

9. Required the area of a trigon, viz. an equilateral triangle, whose side is 20, and perpendicular 5·773502. *Ans.* 173·20506.

10. Required the area of a tetragon, viz. a square whose side is 20, and perpendicular from the centre to the middle of one of the sides 10. *Ans.* 400.

PROBLEM XI.

Given the Diameter of a Circle to find the Circumference, or the Circumference to find the Diameter.

The ratio of the diameter of a circle to its circumference has never yet been *exactly* determined. This Problem has engaged the attention, and exercised the abilities of the greatest mathematicians for ages; no square, or any other right-lined figure, has yet been found, that shall be *perfectly* equal to a given circle. But though the relation between the diameter and circumference has not been accurately expressed in numbers, it may be *approximated* to any assigned degree of exactness. *Archimedes*, about two thousand years ago, discovered the ratio to be nearly as 7 to 22; other, and nearer, ratios have since been successively assigned, *viz.*

As 106 to 333,
As 113 to 355, &c.

This last ratio is very useful, for being turned into a decimal, it agrees with the *truth* to the sixth figure inclusively. *Vieta*, in his *Universalium Inspectionem ad Canonum Mathematicum*, published in 1579, by means of the inscribed and circumscribed polygons of 393216 sides, carried the ratio to ten places of figures, shewing that if the diameter of a circle be 1, the circumference will

be greater than 3·1415926535,
but less than 3·1415926537.

And *Ludolf Van Ceulen*, in his book *De Circulo et Adscriptis*, by the same means carried the ratio to 36 places of figures; this was thought so extraordinary a performance, that the numbers were cut on his *tomb-stone* in St. *Peter's* church-yard, at *Leyden*. These numbers were afterwards confirmed by *Willebrord Snell*. Mr. *Abraham Sharp*, of *Little Horton*, near *Bradford*, *Yorkshire*, extended the ratio to 72 places of figures, by means of Dr. *Halley's* Series, as may be seen in *Sherwin's* Logarithms. Mr. *Machin*, Professor of Astronomy in *Gresham* College, carried the ratio to one hundred places of figures. Lastly, M. *de Lagny*, in the *Memoires de l'Acad.* 1719, by means of the tangent of an arc of 30 degrees, has carried the ratio to the amazing extent of 128 figures; finding that if the diameter be 1, the circumference will be 3·1415, 92653, 58979, 32384, 62643, 38327, 95028, 84197, 16939, 93751, 05820, 97494, 45923, 07816, 40628, 62089, 98628, 03482, 53421, 17067, 98214, 80865, 13272, 30664, 70938, 446 + or 447 —. But on ordinary occasions 3·1416 is generally used, as being sufficiently exact.

There is no figure that affords a greater variety of useful properties than the circle. It is the most capacious of all plane figures, or contains the greatest area within the same perimeter, or has the least perimeter about the same area.

The area of a circle is always less than the area of any regular polygon circumscribed about it, and its circumference always less than the circumference of the polygon. But on the other hand, its area is always greater than that of its inscribed polygon, and its circumference greater than the circumference of its inscribed polygon; hence the circle is always limited between these polygons. The area of a circle is equal to that of a triangle, whose base is equal to the circumference, and perpendicular equal to the radius.

Circles, like other similar plane figures, are in proportion to one another, as the squares of their diameters: and the circumferences of circles, are to one another as their diameters, or radii. *Keith's Geometry*, book vii. prop. 194 and 195, &c.

RULES.

I. As 7 is to 22, so is the diameter to the circumference.

Or, as 113 is to 355, so is the diameter to the circumference.

Or, as 1 is to 3·1416, so is the diameter to the circumference.

II. As 22 is to 7, so is the circumference to the diameter.

Or, as 355 is to 113, so is the circumference to the diameter.

Or, as 3·1416 is to 1, so is the circumference to the diameter; or, which is the same thing, as 1 is to ·318309 *, so is the circumference to the diameter.

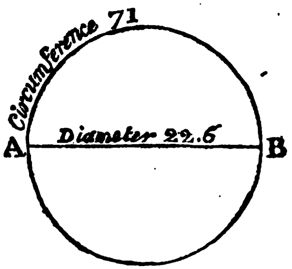
Example 1. The diameter of a circle is 22·6, what is the circumference?

---

* This number is found by dividing 1 by 3·1416.

(1.) 22·6

(1.) 22·6	(2.) 355	(3.) 3·1416
22	22·6	22·6
452	2130	188496
452	710	62832
	710	62832
7)497·2(71.028 *circum.*	113)8023·0(71 *circum.*	71·00016 *circum.*
	791	
	113	
	113	
	...	



2. The circumference of a circle is 71, what is the diameter?

(1.) 71	(2.) 113	(3.) ·318309
7	71	71
22)497(22·59	113	318309
44	791	2228163
57	355)8023(22·6	22·599939
44	710	
130	923	
110	710	
200	2130	
198	2130	
2	...	

By the second and third proportions, the diameter is 22·6, but by the first it falls something short.

3. If the diameter of the earth be 7970 miles, what is its circumference, supposing it a perfect sphere? *Ans.* 25038·552 miles.

4. Required the circumference of a circle, whose diameter is 50 feet. *Ans.* 157·08 feet.

5. If the circumference of a circle be 12, what is the diameter? *Ans.* 3·819708.

PROBLEM XII.

To find the Length of any Arc of a Circle.

Rule I. Multiply the radius of the circle by the number of degrees in the given arc, and that product by ·01745329[*], the last product will be the length of the arc.

Rule II. From eight times the chord of half the arc, subtract the chord of the whole arc, and one-third of the remainder will be the length of the arc *nearly*[†].

Example

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[*] When the radius of a circle is 1, the semi-circumference is 3·14159265, &c. and this number divided by 180, the degrees in a semicircle, gives ·01745329 the length of 1 degree when the radius $= 1$.

[†] This rule was first given by Huygens, a Dutch mathematician. The following approximating theorem, which is Dr. Hutton's, is more accurate: let $d =$ the diameter, and v the versed sine, then $\left(5d\sqrt{\frac{5v}{5d-3v}} + 4\sqrt{dv}\right) \times \frac{2}{9}$ will give the length of the arc.

The following geometrical theorems are useful in mensuration. 1. If a chord AB of an arc ADB be bisected at right angles by the straight line ED, that line is the diameter of the circle (Euclid 1 *of* III. *Corol.*)

2. If a diameter bisects a chord, it cuts it at right angles, (Euclid 3 *of* III.)

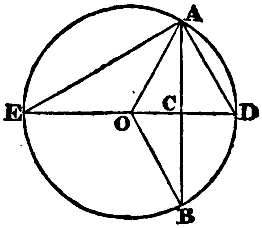
3. A straight line drawn from the centre of a circle to bisect a chord, bisects the arc likewise, (*Keith's* Euclid 3 *of* III, note 17), and cuts the chord at right angles.

4. An

Example 1. If the arc of a circle contain 118 degrees 46 minutes 40 seconds, and the radius of the circle be 19·9855, what is the length of the arc?

First 118 deg. 46 m. 40 sec. $= 118\frac{7}{9}$ degrees; and $19{\cdot}9855 \times 118\frac{7}{9} = 2373{\cdot}8332\frac{7}{9}$, hence $2373{\cdot}8332 \times {\cdot}01745329 = 41{\cdot}431199251228$ the length of the arc.

Example 2. If the chord A D of half the arc A D B be 19·8, and the chord A B of the whole arc 34·4; what is the length of the arc?



$(19{\cdot}8 \times 8) - 34{\cdot}4 = 124$, and $124 \div 3 = 41{\cdot}33\frac{1}{3}$ length of the arc A D B.

---

4. An angle E A D in a semi-circle is a right angle (Euclid 31 of III.)

5. In a right-angled triangle the square of the hypothenuse is equal to the sum of the squares of the other two sides, (Euclid 47 of I.) see also Problem VI.

6. If a perpendicular be drawn from the right angle upon the hypothenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other, (Euclid 8 of VI.)

7. In similar triangles the sides opposite to the equal angles are proportional, (Euclid 4 of VI.) Hence,

$EC : EA :: EA : ED$, or $EC \times ED = EA^2$
$DC : AD :: AD : ED$, or $DC \times ED = AD^2$

8. If two straight lines within a circle cut one another, the rectangle contained by the segments of one of them, is equal to the rectangle contained by the segments of the other, (Euclid 35 of III.) Hence,
$EC \times CD = AC \times CB$, and because $AC = CB$ in this figure, $EC \times CD = AC^2$, or $EC : AC :: AC : CD$, consequently $EC = AC^2 \div CD$, and $CD = AC^2 \div EC$, therefore if the chord of any arc, and its versed sine be given, the diameter is easily found.

9. An angle at the centre of a circle is double to an angle at the circumference, if they stand upon the same arc, (Euclid 20. of III.) Hence the angle A O D is double of the angle A E D.

3. If the chord A B of the whole arc A D B be 6, and the chord A D of half the arc 3·043836, what is the length of the arc? *Ans.* 6·116896.

4. If the radius of the circle be 9, and the arc contain 38·9424412 degrees, what is its length? *Ans.* 6·117063.

5. There is an arc of a circle, whose chord A B is 50·8 inches, and the chord A D 30·6, what is the length of the arc? *Ans.* $64\frac{2}{3}$ inches.

6. The chord A B of the whole arc is 40, and the versed sine D C 15, what is the length of the arc? *Ans.* $53\frac{1}{3}$.

7. If the radius of the circle be 11·3, and the arc contain 52 degrees 15 minutes, what is its length? *Ans.* 10·304858, &c.

8. If the chord A B be 36, and the versed sine C D = 6, what is the diameter E D of the circle? *Ans.* 60.

9. If the chord A B be 18, and the versed sine D C = 4, what is the diameter of E D? *Ans.* $24\frac{1}{4}$.

10. If the chord A B be 12, and the radius O A or O B = 10, what is the length of the arc A D B? *Ans.* 12·86548.

PROBLEM XIII.

To find the Area of a Circle.

RULES.

* 1. Multiply half the circumference by half the diameter: or take $\frac{1}{4}$ of the product of the whole

circumference and diameter, and it will give the area.

---

* 1. Every circle may be considered to be a regular polygon of an infinite number of sides. Now the semi-diameter must be equal to the perpendicular of such a polygon, and the circumference of the circle equal to the sum of the sides of the polygon. Therefore half the circumference, multiplied by half the diameter, gives the area.

2. All circles are to each other as the squares of their diameters (EUCLID 2 *of* XII.) and the area of a circle whose diameter is 1 is ·7854 (Example 2d) $\because 1^2 : d^2 :: \cdot 7854 : d^2 \times \cdot 7854$ the area of a circle whose diameter is *d*.

3. 113

2. Multiply the square of the diameter by ·7854, and the product will be the area.

3. As 452 is to 355, so is the square of the diameter to the area.

---

3. 113 : 355 :: diameter 1 : circumference (Prob. XI.) Now the area of a circle whose diameter is 1 is $\frac{1}{4}$ of this circumference (*Rule* I.) $= \frac{355}{452}$, and the areas of circles are to each other as the squares of their diameters, $\therefore 1^2 : \frac{355}{452} :: d^2 : a =$ the area, viz. $452 : 355 :: d^2 : a$.

4. 7 : 22 :: diameter 1 : circumference (*Prob.* XI), and by the same argument as above, the area of a circle whose diameter is 1 is $\frac{22}{28} = \frac{11}{14}$; hence $14 : 11 :: d^2 : a$.

5. Because the circumferences of circles have the same ratio to each other as their diameters, (*Keith's Geom.* 194 *of* VII.) and the areas are as the squares of the diameters, they are also to each other as the squares of the circumference (EUCLID 11 *of* V), and ·07958 is the area of a circle whose circumference is 1, for when the circumference is 1, the diameter is ·318309.

6. 22 : 7 :: circumf. : diameter, but the area of a circle whose circumference is 1, is $\frac{1}{4}$ of this diameter (*Rule* I.) viz. $\frac{7}{88}$ and circles are to each other as the squares of their circumferences $\therefore 1^2 : \frac{7}{88} :: c^2 : a$, viz. $88 : 7 :: c^2 : a$. where $c =$ the circumference.

7. 355 : 113 :: circumference 1 : diameter, and by the same argument as above, the area of a circle whose circumference is 1, is $\frac{113}{1420}$, $\therefore 1420 : 113 :: c^2 : a$.

Note. If $d =$ the diameter, $c =$ the circumference, $a =$ the area, and $m = 3{\cdot}1416$.

Then 1st. $d = \frac{c}{m} = \frac{4a}{c} = 2\sqrt{\frac{a}{m}}$.

2d. $c = md = \frac{4a}{d} = 2\sqrt{ma}$.

3d. $a = \frac{md^2}{4} = \frac{c^2}{4m} = \frac{dc}{4}$.

4th. $m = \frac{c}{d} = \frac{4a}{d^2} = \frac{c^2}{4d}$.

4. As 14 is to 11, so is the square of the diameter to the area.

5. Multiply the square of the circumference by ·07958, and the product will be the area.

6. As 88 is to 7, so is the square of the circumference to the area.

7. As 1420 is to 113, so is the square of the circumference to the area.

Note. To find the area of a semi-circle, or of a quadrant, find the area of the whole circle, and divide it by 2 or by 4 for the answer.

Example 1. The diameter of a circle is 22·6, and its circumference 71, what is the area by the first Rule?

35·5 half circumf.	22·6 diameter.
11·3 half diameter.	71 circumf.
1065	226
355	1582
355	4)1604·6
401·15 area.	401·15 area.

2. If the diameter of a circle be 1, and its circumference 3·1416, what is the area? *Ans.* ·7854.

3. If the diameter of a circle be 22·6, what is the area by the second Rule? *Ans.* 401·150904.

4. If the diameter of a circle be 22·6, what is the area by the third Rule? *Ans.* 401·15.

5. If the diameter of a circle be 22·6, what is the area by the fourth Rule? *Ans.* 401·31 the area, being larger than by the other methods.

6. If the circumference of a circle be 71, what is the area by Rule 5? *Ans.* 401·16278.

7. If the circumference of a circle be 71, what is the area by the sixth Rule? *Ans.* 400·98.

8. If the circumference of a circle be 71, what is the area by the seventh Rule? *Ans.* [illegible]·15.

9. What is the area of a circle whose diameter is 12? *Ans.* 113·0976.

10. The diameter of a circle is 12, and its circumference 37·6992, what is the area? *Ans.* 113·0976.

11. The circumference of a circle is 37·6992, what is the area? *Ans.* 113·104579, &c.

12. The surveying-wheel is so contrived as to turn just twice in the length of a pole, or 16½ feet; in going round a circular bowling-green, I observed it to turn exactly 200 times; what is the area of the bowling-green? *Ans.* 4 acres 3 roods 35·8 perches.

13. The length of a line, with which my gardener formed a circular fish-pond, was exactly 27¼ yards; pray what quantity of ground did the fish-pond take up? *Ans.* 2419·22835 yards, or half an acre nearly.

14. The diameter of a circle is 30, and the semicircumference 47·124; what is the area of the semicircle? *Ans.* 353·43.

15. The diameter of a circle is 200, what is the area of the semicircle? *Ans.* 15708.

16. If the circumference of a circle be 157·08, and the diameter 50; what is the area of the semicircle? *Ans.* 981·75.

17. Required the area of a quadrant, the radius being 15, and the length of the arc line 23·562. *Ans.* 176·715.

18. The diameter of a circle is 200, what is the area of the quadrant? *Ans.* 7854.

PROBLEM XIV.

Given the Diameter, Circumference, or Area of a Circle; to find the Side of a Square equal in Area to the Circle, and the Side of a Square inscribed in the Circle: or, having the Side of a Square given, to find the Diameter of its circumscribing Circle, and also of a Circle equal in Area, &c.

RULES.

1. The diameter of any circle, multiplied by ·8862269, will give the side of a square equal in area.

2. The circumference of any circle, multiplied by ·2820948, will give the side of a square equal in area.

3. The diameter of any circle, multiplied by ·7071068, will give the side of a square inscribed in that circle.

4. The circumference of any circle, multiplied by ·2250791, will give the side of a square inscribed in that circle.

5. The area of any circle, multiplied by ·6366197, and the square root of the product extracted, will give the side of a square inscribed in that circle.

6. The side of any square multiplied by 1·414236, will give the diameter of its circumscribing circle.

7. The side of any square, multiplied by 4·4428829, will give the circumference of its circumscribing circle.

8. The side of any square, multiplied by 1·1283791, will give the diameter of a circle equal in area to the square.

9. The side of any square, multiplied by 3·5449076, will give the circumference of a circle equal in area to the square.

Example 1. If the diameter of a circle be 22·6, what is the side of a square equal in area to the circle? *Ans.* 20·02872794.

2. If the circumference of a circle be 40, what is the side of a square equal in area to the circle? *Ans.* 11·283792.

3. If the diameter of a circle be 50, what is the side of a square inscribed in that circle? *Ans.* 35·35534.

4. If the circumference of a circle be 40, what is the side of a square inscribed in that circle? *Ans.* 9·003164.

5. If the area of a circle be 1963·5, what is the side of a square inscribed in that circle? *Ans.* 35·3553.

6. If the side of a square be 35·36, what is the diameter of a circle which will circumscribe that square? *Ans.* 50·006592896.

7. If the side of a square be 35·36, what is the circumference of a circle which will circumscribe that square? *Ans.* 157·100339344.

8. If the side of a square be 35·36, what will be the diameter of a circle having the same area as the square? *Ans.* 39·899484976.

9. If the side of a square be 35·36, what is the circumference of a circle, having the same area as the square? *Ans.* 125·347932736.

PROBLEM XV.

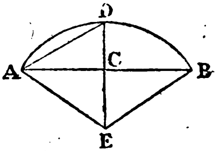
To find the Area of the Sector of a Circle.

Rule I. Multiply half the length of the arc by the radius of the circle, and the product is the area of the sector.

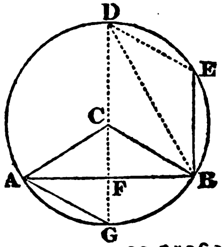
Rule II. As 360 is to the degrees in the arc of the sector, so is the area of the whole circle to the area of the sector.

Example 1. Let ADBE be the sector of a circle less than a semicircle, whose radius AE or BE is 24·5, and the chord AB = 39·2; what is the area?

First, $\sqrt{AE^2 - AC^2} = \sqrt{600{\cdot}25 - 384{\cdot}16} = 14{\cdot}7 = EC$. Then $ED - EC = AE - EC = 24{\cdot}5 - 14{\cdot}7 = 9{\cdot}8 = CD$. And $\sqrt{AC^2 + CD^2} = \sqrt{384{\cdot}16 + 96{\cdot}04} = 21{\cdot}9135 = AD$ the chord of half the arc. Hence, by Rule II. Problem XII. the length of the arc will be found = 45·3693, the half of which, viz. 22·68466, multiplied by 24·5 the radius, produces 555·77417 the area required.



Example 2. Let ADEBCA be a sector, greater than a semicircle, whose radius AC or CB = 20·6, the chord BD of the arc DEB = 35·4, and the chord BE of half that arc = 20·35962. Required the area.



20·35962 = chord BE
8

162·87696
35·4 = chord BD

3)127·37696

42·49232 = arc DEB.

42·49232 = DEB
20·6 = BC

25495392
8498464·

875·341792 area.

3. Required the area of a sector, less than a semicircle, whose radius is 9, and the chord of the arc 6.

Ans. The length of the arc is 6·116896, and area 27·526032.

4. Required the area of a sector, whose arc contains 18 degrees, the radius being 3 feet.

Ans. The length of the arc is ·94247766, and the area 1·41371649.

5. Required the area of a sector, greater than a semicircle, whose radius AC or BC = 10, and the chord BD of half the arc ADEB, viz. the chord of the whole arc DEB = 16.

Ans. As in Example 1, you will find BE = 8·9442719, and the length of the arc DEB 18·518050584; hence the whole area will be 185·18050584.

6. What is the area of a sector AGBC less than a semicircle, the chord of the whole arc AB being 50·8, and the chord of half the arc AG 30·6.

Ans. The length of the arc AGB = $64\frac{2}{3}$, versed sine FG = 17·0645, the diameter DG = 54·8716*, radius CG = 27·4358, and area of the sector 887·0908.

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* Vide *Note* 8, Problem XII.

PROBLEM XVI.

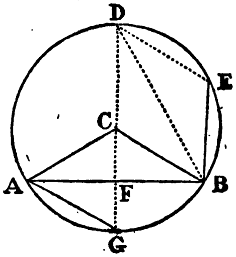
To find the Area of the Segment of a Circle.

RULE I. Find the area of the sector, having the same arc as the segment. (Problem XV.)

Find the area of the triangle, formed by the chord of the segment and the radii of the sector.

Then, if the segment be less than a semicircle, the difference of these two areas will give the answer; but if the segment be greater than a semicircle, take their sum for the area required.

Example 1. Required the area of the segment AGB, less than a semicircle, the chord AB being 35 and the versed sine $FG = 9{\cdot}6$.



First, $FG = 9{\cdot}6 : AF = 17{\cdot}5 :: AF = 17{\cdot}5 : FD = 31{\cdot}9$. Note 8, Prob. XII. hence the diameter $DG = 41{\cdot}5$, the radius $CG = 20{\cdot}75$, and the perpendicular FC of the triangle $ACB = 11{\cdot}15$.

Secondly, $\sqrt{AF^2 + FG^2} = \sqrt{17{\cdot}5^2 + 9{\cdot}6^2} = 19{\cdot}96 = AG$ the chord of half the arc AGB.

11·15 = FC.
35 = AB.
5575
3345
2)390·25
195·125 area of the triangle ACB.

19·96 = AG.
8
159·68
35· = AB.
3)124·68
2)41·56 arc AGB.
20·78 half arc.

20·78 = half arc A G B.
20·75 = radius C G.

Product 431·1850 = area of the sector.
195·125 = area of the triangle.

Remains 236·0600 = area of the segment.

Example 2. Let A D E B F A be a segment greater than a semicircle; there are given the chord A B of the segment = 20·5, the height F D = 17·17, the radius of the circle = 11·64, the chord of half the arc A D E B, viz. B D = 20, and the chord of one-fourth of the arc, viz. B E = 11·5, to find the area of the segment.

$$\frac{(\text{BE} \times 8) - \text{BD}}{3} = \frac{(11{\cdot}5 \times 8) - 20}{3} = 24 \text{ the}$$

length of the arc B E D (Prob. XII.) 24 × 11·64 = 279·36 area of the sector A D E B C A, (Prob. XV).

Now F D — C D = 17·17 — 11·64 = 5·53 = F C, and

$$\frac{\text{AB} \times \text{FC}}{2} = \frac{20{\cdot}5 \times 5{\cdot}53}{2} = 56{\cdot}6825 \text{ area of the}$$

triangle A B C.

Lastly, 279·36 + 56·6825 = 336·0425 the area of the segment required.

RULE II*. To the square of half the chord of the segment add the square of the versed sine, the square

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* This Rule is Sir I. Newton's. The following Rule, which is likewise his, is something shorter, though not so accurate.

RULE III. To the square of half the chord of the segment add the square of the versed sine, the square root of this sum will give the chord of half the arc of the segment.

To $\frac{4}{3}$ of the chord of half the arc of the segment add the chord of the segment, the sum multiplied by $\frac{2}{5}$ of the versed sine will give the area.

To these may be added the following Rule, which is simple in its application, and sufficiently accurate for practical purposes, when the segment is not very large.

RULE IV. To two thirds of the product of the chord and height square root of this sum will give the chord of half the arc of the segment.

To the square of the chord of the segment add the square of the versed sine, and to twice the square root of this sum add the chord of half the arc; multiply the result by the versed sine, and $\frac{4}{15}$ of the product will give the area.

Example 3. Required the area of a segment A G B less than a semicircle, whose chord A B is 35 and versed sine F G $= 9{\cdot}6$.

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height of the segment, add the cube of the height divided by twice the chord, and the sum will be the area of the segment, nearly.

When the segment is greater than a semicircle, find the area of the remaining segment, which subtract from the area of the whole circle.

Note. This Rule has been inserted in several books, but hitherto without demonstration. Mr *Peter Nicholson*, in a letter to the editor of this work, written from *Carlisle*, sometime ago, states that the Rule was *originally* given by him.

Demonstration. Let $v =$ the versed sine F G of the segment A G B, and $d =$ D G the diameter of the circle, then by Dr. *Hutton's Mensuration*, p. 101, quarto edition, published in 1770, the area of the segment $= 2v\sqrt{dv} \times \left(\frac{2}{3} - \frac{v}{5d} - \frac{v^2}{28d^2} - \frac{v^3}{72d^3}, \&c.\right)$ But (by Note 8, Prob. XII. of this Treatise) D F × F G = A F², or $(d-v) \times v = \frac{c^2}{4}$, hence $d = \frac{c^2}{4v} + v$ and $2v\sqrt{dv} = v\sqrt{c^2+4v^2} = vc + \frac{2v^3}{c} - \frac{2v^5}{c^3} + \frac{4v^7}{c^5} - \&c.$ $\therefore \left(vc + \frac{2v^3}{c}\right) \times \left(\frac{2}{3} - \frac{v}{5d}\right) = \frac{2}{3}vc + \frac{4v^3}{3c} - \frac{vc^2}{5d}, \&c. = \frac{2}{3}vc + \frac{4v^3}{3c} - \frac{4vc^3}{5c^2+20v^2} = \frac{2}{3}vc + \frac{8c^2v^3+80v^5}{15c^3+60vc} = \frac{2}{3}vc + \frac{8v^3}{15c} - \frac{32v^4+80v^5}{15c^3+60vc} = \frac{2}{3}vc + \frac{8v^3}{15c} = \frac{2}{3}vc + \frac{v^3}{2c}$ nearly, which is the rule, $= \left(\frac{2}{3}c + \frac{v^2}{2c}\right) \times v$, where c is the chord A B of the segment.

First, $\sqrt{AF^2 + FG^2} = \sqrt{17{\cdot}5^2 + 9{\cdot}6^2} = 19{\cdot}96 =$ A G the chord of half the arc A G B.

Secondly, $\sqrt{AB^2 + FG^2} = \sqrt{35^2 \times 9{\cdot}6^2} = 36{\cdot}2926$.

Then $(\overline{36{\cdot}2926 \times 2} + 19{\cdot}96) \times 9{\cdot}6 \times \frac{4}{15} = 236{\cdot}9157$ area of the segment. Vide Example 1.

4. The area of a segment A D E B F A, greater than a semicircle is required, the chord A B being 20·5, and height F D 17·17.

By Note 8, Prob. XII. the height F G of the remaining segment A G B = 6·119: hence the diameter is 23·289, and the area of the whole circle 425·983305. By Rule II. the area of the segment A G B is 89·29644, which, subtracted from the area of the whole circle, leave 336·68686 for the area required. See Example 2.

5. Required the area of a segment A G B, less than a semicircle, whose chord A B is 18·9, and height F G = 2·4.

$$\textit{Ans.}\begin{cases} 30{\cdot}601875 \text{ by Rule 1.} \\ 30{\cdot}62626 \text{ by Rule 2.} \end{cases}$$

6. Required the area of a segment A G B, less than a semicircle, whose chord A B is 23, and versed sine F G = 3·5.

$$\textit{Ans.}\begin{cases} 54{\cdot}5844 \text{ by Rule 1.} \\ 54{\cdot}647 \text{ by Rule 2.} \end{cases}$$

7. Required the area of a segment A D E B F A greater than a semicircle, whose chord A B is 12 and versed sine F D = 18.

Ans. The diameter D G of the circle = 20, and its area = 314·16. The versed sine F G of the smaller segment A G B = 2, and its area, by Rule 2, = 16·34965, hence the area of the whole segment A D E B F A = 297·81034.

PRO-

PROBLEM XVII.

To find the Area of the Segment of a Circle by the Table given at the End of this Treatise.

Rule. Divide the height of the segment by the diameter of the circle, to three places of decimals.

Find the quotient in the column *height*, and take out the corresponding *area seg.* which multiply by the square of the diameter, and the product will be the area of the segment required.

Note. If the quotient of the height by the diameter be greater than ·5, subtract it from an unit, and find the *Area Seg.* corresponding to the remainder, which subtract from ·7854 for the correct *Area Seg.*

If the quotient of the height by the diameter does not terminate in three figures, find the *Area Seg.* answering to the first three decimals of the quotient, subtract it from the next greater *Area Seg.* multiply the remainder by the fractional part of the quotient, and add the product to the *Area Seg.* first taken out of the table.

Example 1. Required the area of the segment of a circle, whose height is 3·25, the diameter of the circle being 50.

3·25 ÷ 50 = ·065 the tabular height, against which stands the *Area Seg.* ·021659; hence ·021659 $\times$ 50^2 = 54·1475, the area required.

2. Required the area of the segment of a circle, whose height is 46·75, and the diameter of the circle 50.

46·75 ÷ 50 = ·935, hence 1 — ·935 = ·065 tabular height, against which stands the *Area Seg.* ·021659, then ·7854 — ·021659 = ·763741, the correct *Area Seg.*; hence ·763741 $\times$ 50^2 = 1909·3525 the area required.

3. Required the area of the segment of a circle, whose height is 2, and the diameter of the circle 52.

2 ÷ 52 = ·038 $\frac{6}{13}$ the tabular height. The *Area Seg.* answering to ·038 is ·009763, the next greater *Area Seg.* is ·010648, the difference is ·000385; $\frac{6}{13}$ of

which is ·000177, then ·009763 + ·000177 = ·009940 the correct *Area Seg.* corresponding to ·038$\frac{6}{13}$, hence ·009940 × 52^2 = 26·87776, the area required.

4. Required the area of a segment, whose chord is 12, and height 18.

Ans. The diameter of the circle = 20, and the area of the segment = 297·81.

5. The chord of a circular segment is 20, and the height 5, what is the area?

Ans. The diameter of the circle = 25, and the area of the segment = 69·889375.

PROBLEM XVIII.

To find the Area of a Circular Zone, or the Space included by two parallel Chords, and the Arcs contained between them.

Rule. Join the extremities of the parallel chords by two straight lines, so as to form a trapezoid; then the two opposite segments which these lines cut off will be equal[*].

To the area of the trapezoid add twice the area of one of the segments, and the sum will be the area of the zone.

Example 1. Suppose the greater chord AB = 40, the less chord CD = 30, and their perpendicular distance Dm = 35[†], what is the area of the zone ABDC?

First

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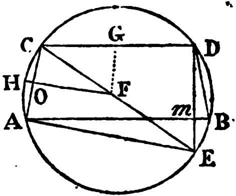
[*] *Keith's Geom.* 165 of VII.

[†] General Construction. Draw AB of the given length, make mB equal to half the difference between AB and CD, erect the perpendicular mD, and make it equal to the distance between the given chords, through D draw DC of the given length, parallel to AB, and join AC.

Bisect AC and DC by the perpendiculars OF and GF, then F will be the centre of the circle, join FC and describe the circle, with F as a centre, and FC as a radius.

Produce



First $\frac{1}{2}(AB - CD) = mB = \frac{1}{2}(40 - 30) = 5$. $\sqrt{Dm^2 + mB^2} = DB = \sqrt{35^2 + 5^2} = 35{\cdot}35534$ $AB - mB = Am = 40 - 5 = 35$, and $(Am \times Bm) \div Dm = Rm = (35 \times 5) \div 35 = 5$; $Dm + Em = DE = 35 + 5 = 40{\cdot}$ $\frac{1}{2}\sqrt{CD^2 + DE^2} = \frac{1}{2}CE = FH = \frac{1}{2}\sqrt{30^2 + 40^2} = 25$ the radius of the circle. $(DB \times Am) \div 2Dm = OF = (35{\cdot}35534 \times 35) \div (35 \times 2) = 17{\cdot}67707$, $FH - OF = OH = 25 - 17{\cdot}67767 = 7{\cdot}32233$ height of the segment AHC.

$7{\cdot}3223 \div 50 = {\cdot}146446$, the tabular *Area Seg.* corresponding to which is ·071348, hence $\cdot071348 \times 50^2 = 178{\cdot}37$ the area of the segment AGC. $\frac{1}{2}(AB + CD) \times Bm = \frac{1}{2}(40 + 30) \times 35 = 1225$ area of the trapezoid ABDC.

Produce Dm to E, and join FE; then because Dm is perpendicular to AB, it is also perpendicular to CD, (*Keith's Geom.* 27 of VIII.) therefore the angle CDE is a right angle, and CE is a straight line and a diameter of the circle. Join AE, then the angle CAE is a right angle, (EUCLID 31 of III.) and because EF is the half of CE, and CO the half of CA, OF is parallel to and equal the half of AE, (*Keith's Geom.* 62 of VII.)

GENERAL METHOD OF SOLUTION. $\frac{1}{2}(AB - CD) = mB$; $\sqrt{Dm^2 + mB^2} = DB$ (EUCLID 47 of I.) $AB - mB = Am$, and $Am \times mB = Dm \times mE$, (EUCLID 35 of III.) therefore $Em = (Am \times Rm) \div Dm$. Now $Dm + Em = DE$, and $\sqrt{CD^2 + DE^2} = CE$ *the diameter of the circle*, the half of which gives $EF = FH$.

The triangles EmA and DmB are equiangular, for they are right angled at m, and the angles EAB and EDB are equal, because they both stand in the same arc BE, (EUCLID 21 of III.) therefore,

$Dm : DB :: Am : AE$, the half of which is OF, viz. $OF = (DB \times Am) \div 2Dm$.

Lastly, $FH - FO = OH$ the height of the segment AHC.

Note. If the diameter of the circle be given, the operation will be much shorter; or if the chord AC and height OH be given, the operation will be very simple, and in this case the distance mD need not be given.

Lastly, 1225 + (178·37 × 2) = 1581·74 the area of the zone required.

2. Suppose the greater chord AB = 96, the less chord CD = 60, and the distance between them Dm = 26, what is the area of the zone ABDC?

Ans. The diameter CE = 100, height of the segment AHC = 2·5658, area = 54·38 by the table of circular segments. Area of the zone ABDC = 2136·76.

3. Suppose the greater chord AB = 48, the less chord CD = 30, and the chord AC of the segment AHC = 15·8114, what is the area of the zone ABDC?

Ans. The diameter CE = 50, height of the segment AHC = 1·2829, area = 13·595 by the table of circular segments. Area of the zone ABDC = 634·19.

4. Required the area of the compound figure AEBDCHA, the chord ED being 23, and height of the segment EBD 3·5, the chord AC = 18·9, and height of the segment AHO = 2·4; also the diagonal CE of the trapezium AEDC = 34, and a perpendicular from D upon it = 16·2, and from A = 16.

Ans. The area of the segment AHC = 30·62626, (*Example* 4, Prob. XVI.) The area of the segment EBD = 54·647, (*Example* 5, Prob. XVI.) area of the trapezium = 547·4, area of the whole compound figure = 632·67326.

PROBLEM XIX.

To find the Area of a circular Ring, or of the Space included between two concentric Circles.

RULE *. Multiply the sum of the diameters of the two circles by their difference, and that product by ·7854, the last product will give the area of the ring.

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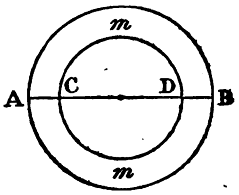
* Let D = the diameter of the larger circle, and d = that of the smaller, then will D^2 × ·7854 be the area of the former, and

Note. The area of part of a ring, or of the segment of a sector, may be found by multiplying half the sum of the bounding arcs by their distance[*].

Example 1. The diameters AB and CD of two concentric circles are 20 and 15, what is the area of the ring m, m, contained between their circumferences?



$(AB + CD) \times (AB - CD) \times ·7854 = 35 \times 5 \times ·7854 = 137·445$ the area of the ring m, m.

2. The diameters of two concentric circles are 24 and 18, what is the area of the ring contained between their circumferences? *Ans.* 197·9208.

3. The inner diameter of a circular light-house is 25 feet, and the thickness of the wall is 3 feet 6

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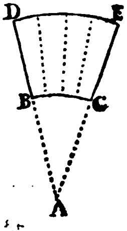
and $d^2 \times ·7854$ the area of the latter. The difference of these two areas will, evidently, be the area of the ring, hence $·7854\,D^2 - 7854\,d^2 = (D^2 - d^2) \times ·7854 = (D + d) \times (D - d) \times ·7854$, which is the rule.

[*] Let DE the greater arc $= a$, BC the less arc $= b$, and AD the radius $= r$. Now because similar arcs are to each other as their radii DE : BC :: AD : AB, viz. $a : b :: r : \frac{br}{a}$.



But $AD - AB = BD = r - \frac{br}{a} = \frac{a - b}{a} \times r$. The area of the sector ADE $= \frac{ar}{2}$, and the area of the sector ABC $= \frac{1}{2} b \times \frac{br}{a} = \frac{b^2 r}{2a}$, hence the area of the segment

$$BCED = \frac{ar}{2} - \frac{b^2 r}{a} = \frac{a^2 r - b^2 r}{2a} = \frac{a^2 - b^2}{2a} = \frac{a + b}{2} \times \frac{a - b}{a} \times r = \frac{a + b}{2} \times BD,$$ which is the rule.

inches; how many square feet of ground does the wall stand upon? *Ans.* 313·4746 feet.

4. A circular fish-pond is 50 yards in diameter, a gravel walk of 6 feet wide, and 3 feet from the edge of the water, is to be made round it; what will be the expence of gravel for the walk at 5 shillings per load, supposing every square yard to take half a load of gravel? £42-8-2¼.136 *Ans.* ~~£62. 8. 9¼. ·728.~~

5. What is the area of the part of a circular arch, built with stones, the upper bounding arc being 37 feet 6 inches, the lower 36 feet 4 inches, and the stones each 2 feet 6 inches long?

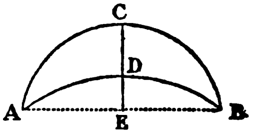
Ans. 92 feet, 3 inches, 6 parts.

PROBLEM XX.

*To find the Area of a Lune**, *or of the Space included between the intersecting Arcs of two Eccentric Circles.*

RULE. Find the areas of the two segments which form the lune, and their difference will be the area required.

Example 1. The length of the chord A B is 60, and the heights E C = 20, and E D = 5, required the area of the lune A D B C A.



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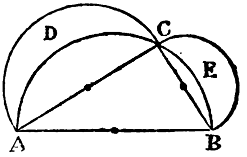
* Several properties of lunes may be seen in *Dr. Rees's New Cyclopedia*; the following is the most important.

A semicircle, described on the hypothenuse of a right-angled triangle, is equal to the two semicircles described on the other two sides; and if the semicircle on the hypothenuse be drawn so as to meet each of the other semicircles in two points, the two lunes, thus formed, will be equal to the triangle: viz. the lunes D and E are equal to the triangle A C B. *Keith's Geometry*, 197 of VII.



By Note 8, Prob. XII. $(AE^2 \div EC) + EC =$ the diameter of the circle of which ACB is an arc, and $(AE^2 \div ED) + ED =$ the diameter of the circle of which ADB is an arc, viz. $(30^2 \div 20) + 20 = 65$, and $(30^2 \div 5) + 5 = 185$.

The $20 \div 65 = ·307\frac{9}{13}$, the *Area Seg.* to which $=$ ·205243, hence ·205243 $\times 65^2 =$ 867·15167 area of the segment ACB; and $5 \div 185 = ·027\frac{1}{37}$ the *Area Seg.* to which $=$ ·005876, hence ·005876 $\times 185^2 =$ 201·1061 area of the segment ADB.

Lastly, 867·15167 — 201·1061 = 666·04557 area of the lune.

By Rule IV. *Note to Prob.* XVI.

$(60 \times 20 \times \frac{2}{3}) + (20^3 \div \overline{60 \times 2}) =$ 866·666 area of segment ACB.

$(60 \times 5 \times \frac{2}{3}) + (5^3 \div \overline{60 \times 2}) =$ 201·041 area of segment ADB.

Area of the lune = 665·625.

By Rule II. *Prob.* XVI.

The area of the greater segment - = 866·9153
The area of the less segment - - = 201·1063

Area of the lune - - - - - - - = 665·8090

2. The chord AB is 40, and the heights of the segments EC and ED are 15 and 2; what is the area of the lune? *Ans.* 388·47384, (*by Prob.* XVI.)

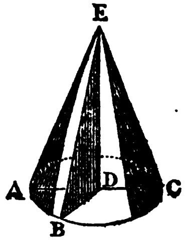
3. The length of the chord AB is 24, and the heights of the segments 9 and 4, what is the area of the lune? *Ans.* 93·69375, (*by Prob.* XVII.)

THE

MENSURATION OF THE CONIC SECTIONS.

1. Conic Sections are such surfaces as are formed by cutting a solid cone by a plane. They are five in number, viz. the triangle, circle, ellipsis, parabola, and hyperbola.

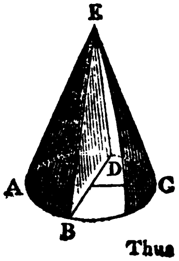
2. A Cone is a solid, generated by the revolution of a straight line about the circumference of a circle, one end of which passes through a fixed point above the plane of that circle; and this fixed point is called the vertex of the cone.



Thus, if the straight line E B, passing through the point E, revolve round the circle A B C in the direction C B A, the solid figure A E C B will be a cone, of which E is the vertex, and E D the axis.

Or, according to Euclid, a cone is formed by the revolution of a right angled triangle about one of its sides, which remains fixed: viz. the cone is formed by the revolution of the right-angled triangle B D E about the fixed line D E, the point B describing the circular base.

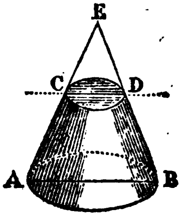
3. If the cutting plane pass through the vertex of the cone and the base, the section will be an isosceles triangle, having its sides each equal to the slant height of the cone, and its base a chord, or diameter, of the circular base of the cone.



Thus if E be the vertex of the cone, the sides B E and D E, of the isosceles triangle B D E, are each equal to A E, the slant height of the cone; and B D, the base of the triangle, is a chord, or diameter, of the circular base A B C D of the cone.

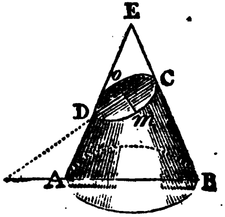
4. If a cone be cut by a plane parallel to the base, the section will be a circle.

Thus, if the cone A B E, be cut by a plane parallel to A B, the section will be a circle, of which C D is the diameter.



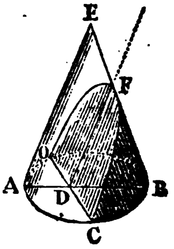
5. If a cone be cut by a plane inclined to the base, so as not to cut the base, the section will be an *Ellipsis*.

Thus, if the cone A B E be cut by a plane inclined to the base, but not cutting it, the section will be an ellipsis, of which D C is the *Axis Major*, and O M the *Axis Minor*. The definitions of the terms belonging to the ellipsis are given at page 6, and the manner of constructing it, at Prob. XXX. p. 22.

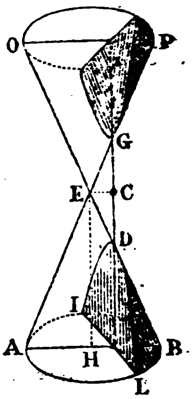


6. If a cone be cut by a plane parallel to one of the slant sides, the section will be a *parabola*.

Thus, if the cone A B E be cut by a plane parallel to the slant side A E, the section O F C will be a parabola, F its vertex, F D the axis, and O C its base. The definitions of the terms belonging to the parabola are given at p. 7, and the manner of constructing it, at Prob. XXXI. p. 23.



7. If a cone be cut by a plane parallel to the axis, or in such a manner that the cutting plane may meet the opposite cone, continued above the vertex, the section will be an *hyperbola.*



Thus, if the cone A B E be cut by a plane parallel to the axis E H, or in such a manner as to meet the opposite cone O P E, the section I D L will be an hyperbola, of which D is the vertex, D G the transverse axis, C the centre, and G the vertex of the opposite cone. The definitions of the terms belonging to the hyperbola are given at p: 8, and the manner of constructing it, at Prob. XXXII. p. 24.

Note. Though there be five different surfaces formed by cutting a cone, yet it is customary to apply the term CONIC SECTIONS only to three of these figures, viz. the *Ellipsis, Parabola*, and *Hyperbola.*

1. OF THE ELLIPSIS.

Some general Properties of the Ellipsis.

1. The sum of two straight lines, drawn from the foci, to meet in any point of the curve, are equal to the transverse axis; and a straight line drawn from one extremity of the conjugate axis to the focus, is equal to half the transverse.

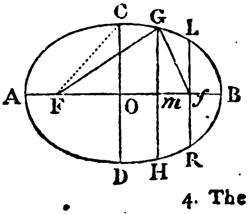
Viz. $FG + Gf = AB$, and $CF = AO$.

2. The distance between the two foci is a mean proportional between the sum and difference of the transverse and conjugate axis.

Viz. $AB + CD : Ff :: Ff : AB - CD$.

3. The square of the distance of the focus from the centre, is equal to the difference of the squares of the semi-axes.

Viz. $FO^2 = AO^2 - CO^2$.



4. The rectangle of the distance of the focus from each vertex, is equal to the square of the semi-conjugate.

Viz. $AF \times FB = CO^2$.

5. The latus rectum, or double ordinate at the focus, is equal to the square of the conjugate divided by the transverse.

Viz. $(CD^2 \div AB) = LR$, or $AB : CD :: CD : LR$.

6. The square of the transverse axis, is to the square of the conjugate; as the rectangle of the two abscissas, is to the square of the ordinate.

Viz. $AB^2 : CD^2 :: Am \times mB : Gm^2$; hence, if $AB = t$, $CD = c$, $Am = x$, $mB = t - x$, and $Gm = y$, then $t^2 : c^2 :: x \times (t - x) : y$; and $y = \frac{c}{t} \sqrt{x \times (t - x)}$, any three of these quantities being given, the fourth may be found from this equation.

7. The transverse axis is to its latus rectum, as the rectangle of the two abscissas is to the square of the ordinate.

Viz. $AB : LR :: Am \times mB : Gm^2$.

8. The square of the semi-transverse is to the rectangle of the distance of the focus from each vertex, as the rectangle of the two abscissas is to the square of the ordinate.

Viz. $AO^2 : AF \times FB :: Am \times mB : Gm^2$.

9. The transverse axis is to its conjugate, as the area of a circle, whose diameter is the transverse, is to the area of the ellipsis.

10. Every parallelogram, circumscribing an ellipsis, is equal to a rectangle formed by the transverse and conjugate axes.

11. The area of an ellipsis is to the rectangle of its two axes, as the area of any circle is to the square of its diameter.

12. The transverse axis is to the conjugate, as the area of a circle, whose diameter is the transverse, is to the area of the ellipsis.

13. The areas of ellipses are to each other, as the rectangles of their transverse and conjugate axes.

14. The square root of the product of the transverse and conjugate diameters, will be the diameter of a circle equal in area to the ellipsis.

Note. All these properties, and many others, are demonstrated in *Emerson's* Conic Sections.

PRO-

PROBLEM XXI.

The transverse and conjugate Diameters of an Ellipsis being given, to find the Area.

Rule. Multiply the transverse diameter by the conjugate, and that product by ·7854, and the result will be the area.

Example 1. Required the area of an ellipsis, whose transverse diameter A B is 61·6, and the conjugate C D = 44·4.

61·6 × 44·4 × ·7854 = 2148 ·100416. *Answer.*

2. The longer diameter of an ellipsis is 50, and the shorter 40, what is the area? *Ans.* 1570·8.

3. The longer diameter of an ellipsis is 70, and the shorter 50, what is the area? *Ans.* 2748·9.

4. Required the area of an ellipsis, whose transverse diameter is 24, and conjugate 18.
Ans. 339·2928.

PROBLEM XXII.

To find the Area of an Elliptical Ring, or the Space included between the Circumferences of two concentric and similar Ellipses *.

Rule. Find the area of each ellipsis, and subtract the less from the greater, the remainder will be the area of the ring.

Or, from the product of the two diameters of the greater ellipsis, subtract the product of the two diameters of the less; the remainder multiplied by ·7854, will be the area of the ring.

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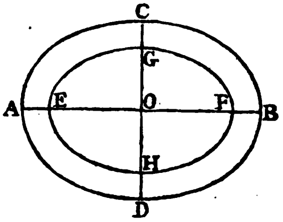
* It is here supposed, that the difference between the conjugate diameters, is equal to the difference between the transverse diameters; but it is well known, that in this case the elliptic arcs will not be every where equi-distant: the difference between the semi-transverse, or semi-conjugate diameters, being the least distance between the arcs.



Example 1. The transverse diameter A B of an ellipsis is 60, the conjugate C D 47; and the transverse diameter E F of another ellipsis, having the same centre O, is 45, and the conjugate G H 32; required the area of the space contained between their circumferences.

47 = C D.	32 = G H.
60 = A B.	45 = E F.
2820 Product	160
1440	128
1380 Remainder.	1440 Product.

Then 1380 × ·7854 = 1083·8520 area of the ring.

2. The ellipsis in *Grosvenor-square* measures 840 links the longest way, and 612 the shortest, within the wall; the wall is 14 inches thick, what quantity of ground does it enclose, and how much does it stand upon?

Ans. { The wall encloses 4 acres, 0 roods, 6·013 p.
And stands on 195·6103 yards.

3. A gentleman has an elliptical fountain in his garden, whose greater diameter is 30, and less 24 feet; and has ordered a walk to be paved round it of 5 feet 6 inches in width, with *Purbeck* stone, at 4 shillings per square yard, what will the expence be?

Ans. { The area of the walk is 561·561 feet.
Expence £12 : 9s. 6¼d. ·968.

PRO-

PROBLEM XXIII.

Given the Height of an Elliptical Segment, whose Base is parallel to either of the Axes of the Ellipsis, and the two Axes to find the Area.

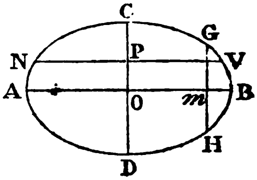
Rule I. Subtract the height of the segment, from that diameter of which it is a part; multiply the remainder by the height of the segment, and twice the square root of the product will be the chord of a circular segment, having the same height.

2. Find the area of this circular segment by Rule II, Problem XVI.

3. Then as that diameter of the ellipsis, of which the height of the segment is a part, is to the other diameter; so is the circular area, to the elliptical area.

Rule II. Divide the height of the segment by that diameter of which it is a part, to three places of decimals, find the quotient in the column *height* in the table, and take out the corresponding *Area Seg.* (See Prob. XVII.)

Multiply the *Area Seg.* thus found, and the two axes of the ellipsis together, and the result will give the area required.



Example 1. Suppose the longer axis AB of an ellipsis to be 120, the shorter axis CD = 40, what is the area of the segment GVBH, whose base G*m*H is parallel to the shorter axis, and its height *m*B = 4?

By Rule I. $(AB - mB) \times mB = (120 - 4) \times 4 = 464$ the square of half the chord of a *circular* segment whose height is 4, and $\sqrt{464} \times 2 = 21{\cdot}5406592 \times 2 = 43{\cdot}0813184$ the chord of the *circular* segment.

Then, by Rule II, *Problem* XVI.

$\sqrt{464 + 4^2} = 21\cdot9089$ chord of half the arc of the circular segment.

Then $2\sqrt{43\cdot0813184)^2 + 4^2} + 21\cdot9089 = 2\sqrt{1856 + 16} + 21\cdot9089 = 108\cdot4421$, and $108\cdot4421 \times 4 \times \frac{4}{15} = 115\cdot67157$, area of the circular segment.

Lastly, $120 : 40 :: 115\cdot67157 : 38\cdot55719$ the area of the elliptic segment G V B H.

By Rule II. *of this Problem.*

$4 \div 120 = \cdot033\frac{1}{3}$, the *Area Seg.* corresponding to which is $\cdot008033$, then $\cdot008033 \times 120 \times 40 = 38\cdot5584$, area of the elliptic segment G V B H.

2. Suppose the longer axis A B of an ellipsis to be 120, the shorter axis C D = 40, what is the area of the segment N C G V, whose base N P V is parallel to the longer axis, and its height P C = 4?

Ans. { 196·1958 by Rule I.
{ 196·2 by Rule II.

3. Required the area of an elliptical segment cut off by a chord or double ordinate, parallel to the shorter diameter, the height of the segment being 10, and the two diameters 35 and 25?

Ans. { 161·9824 by Rule I.
{ 162·02025 by Rule II.

4. What is the area of an elliptical segment cut off by a chord parallel to the longer diameter, the height of the segment being 5, and the two diameters 35 and 25?

Ans. { 97·8346 by Rule I.
{ 97·845125 by Rule II.

5. What is the area of an elliptical segment cut off by a chord parallel to the shorter diameter, the height being 20, and the two diameters 70 and 50?

Ans. { 647·9297 by Rule I.
{ 648·081 by Rule II.

PROBLEM XXIV.

Given the two Diameters of an Ellipsis to find the Circumference.

RULE I. Multiply the sum of the two diameters by 1·5708, and the product will be the circumference; *near enough for most practical purposes.*

RULE II. To half the sum of the two diameters add the square root of half the sum of their squares; multiply this last sum by 1·5708, and the product will be the circumference, *very nearly.*

RULE III. Divide the square of the conjugate axis by the square of the transverse, and subtract the quotient from an unit, call the remainder d. Find the value of the following series *.

$$1 - \left(\frac{d}{2\cdot2}\,(\text{A}) + \frac{3\,d}{4\cdot4}\,\text{A}\,(\text{B}) + \frac{3\cdot5\,d}{6\cdot6}\,\text{B}\,(\text{C}) + \frac{5\cdot7\,d}{8\cdot8}\,\text{C}\,(\text{D}) + \frac{7\cdot9\,d}{10\cdot10}\,\text{D}\,(\text{E}) + \frac{9\cdot11\,d}{12\cdot12}\,\text{E}\,(\text{F}) + \frac{11\cdot13\,d}{14\cdot14}\,\text{F}\,(\text{G}) + \frac{13\cdot15\,d}{16\cdot16}\,\text{G}\,(\text{H}), \&\text{c.}\right)$$

Multiply the value of this series by the transverse axis, and that product by 3·1416, and the result will give the circumference of the ellipsis.

Example 1. The two diameters of an ellipsis are 24 and 18, what is the circumference?

By Rule I.

$(24 + 18) \times 1\cdot5708 = 65\cdot9736$ the circumference, nearly.

By Rule II.

$\frac{1}{2}(24 + 18) + \sqrt{\frac{1}{2}(24^2 + 18^2)} = 21 + \sqrt{450} = 42\cdot2132034$, which multiplied by 1·5708, gives 66·3085 the circumference.

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* Holliday's Fluxions, Prob. X. p. 272.

By Rule III.

$1 - (18^2 \div 24^2) = 1 - {\cdot}5625 = {\cdot}4375 = d.$

The second term (A) $= \dfrac{d}{2 \cdot 2} \qquad = {\cdot}109375$

The third term (B) $= \dfrac{3\,d}{4 \cdot 4}\ \text{A} = {\cdot}008972$

The fourth term (C) $= \dfrac{3 \cdot 5\,d}{6 \cdot 6}\ \text{B} = {\cdot}001635$

The fifth term (D) $= \dfrac{5 \cdot 7\,d}{8 \cdot 8}\ \text{C} = {\cdot}000391$

The sixth term (E) $= \dfrac{7 \cdot 9\,d}{10 \cdot 10}\ \text{D} = {\cdot}000108$

The seventh term (F) $= \dfrac{9 \cdot 11\,d}{12 \cdot 12}\ \text{E} = {\cdot}000032$

The eighth term (G) $= \dfrac{11 \cdot 13\,d}{14 \cdot 14}\ \text{F} = {\cdot}000010$

Sum ·120523

Then $1 - {\cdot}120523 = {\cdot}879477$ value of the series, hence ${\cdot}879477 \times 24 \times 3{\cdot}1416 = 66{\cdot}31115$ the circumference of the ellipsis.

2. What is the circumference of an ellipsis whose transverse axis is 50, and conjugate 40 yards?

Ans. $\begin{cases} 141{\cdot}372 \text{ by Rule I.} \\ 141{\cdot}8069948 \text{ by Rule II.} \end{cases}$

3. The longer diameter of an ellipsis is 70, and the shorter 50, what is the circumference?

Ans. $\begin{cases} 188{\cdot}496 \text{ by Rule I.} \\ 189{\cdot}796 \text{ by Rule II.} \end{cases}$

PROBLEM XXV.

Any three of the four following Quantities being given to find the fourth; viz. an Ordinate, and its Abscissas, and the two Axes[*].

CASE I.

When the transverse, conjugate, and abscissas are given, to find the ordinate.

Rule. The transverse is to the conjugate, as the square root of the rectangle of the two abscissas is to the ordinate.

Example 1. In the ellipsis A D B C, the transverse diameter $AB = 120$, the conjugate $CD = 40$, and the abscissa $mB = 4$, what is the length of the ordinate Gm?

$120 : 40 :: \sqrt{116 \times 4} : 7{\cdot}1802$ the ordinate Gm.

2. When the transverse axis is 80, the conjugate 60, and the abscissa 16, what is the length of the ordinate? *Ans.* 24.

CASE II.

When the transverse axis, conjugate, and ordinate, are given to find the abscissas.

Rule. From the square of the conjugate axis subtract four times the square of the ordinate, and extract the square root of the remainder.

Then, the conjugate axis is to the transverse, as half the aforesaid square root is to the distance of the ordinate from the centre of the ellipsis: this distance, added to half the transverse axis, gives the greater abscissa, and subtracted from it, leaves the less.

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[*] The four following rules are all deduced from the 6th property of the ellipsis, p. 69, viz. $t^2 : c^2 :: x \times (t-x) : y^2$.

Example 1. When the transverse axis A B, of the ellipsis A D B C, is 52, the conjugate C D 26, and the ordinate G *m* 12, what are the two abscissas A *m* and B *m*?

$\sqrt{26^2 - (12^2 \times 4)} = \sqrt{100} = 10$, then $26 : 52 :: 5 : 10 = o\,m$ the distance of the ordinate from the centre o of the ellipsis, then $26 + 10 = 36 = A\,m$, and $26 - 10 = 16 = B\,m$.

2. When the transverse axis is 80, the conjugate 60, and the ordinate 24; what are the two abscissas? *Ans.* 64 and 16.

CASE III.

When the conjugate axis, ordinate, and abscissas, are given, to find the transverse.

Rule. Find the square root of the difference of the squares of the semi-conjugate axis and the ordinate, which add to the semi-conjugate, if the less abscissa be given, but subtract it if the greater abscissa be given.

Then, the square of the ordinate is to the rectangle of the conjugate axis and the abscissa; as the sum, or difference found above, is to the transverse required.

Example 1. In the ellipsis A D B C there are given the conjugate axis $C\,D = 26$, the ordinate $G\,m = 12$, and the less abscissa $B\,m = 16$, required the transverse axis A B.

$\sqrt{13^2 - 12^2} = \sqrt{25} = 5$, then $13 + 5 = 18$, *sum.*

Then $12^2 : 26 \times 16 :: 18 : 52$, the transverse axis A B.

2. If the conjugate axis be 60, the ordinate 24, and the greater abscissa 64; what is the length of the transverse axis? *Ans.* 80.

CASE IV.

When the transverse axis, ordinate, and abscissas, are given to find the conjugate axis.

Rule. The square root of the product of the two abscissas is to the ordinate, as the transverse axis is to the conjugate.

Example 1. In the ellipsis A D B C, there are given the transverse axis A B = 120, the ordinate G m = 16, and the abscissa m B = 24, to find the conjugate axis C D.

120 — 24 = 96 the abscissa A m.

Then $\sqrt{96 \times 24} : 16 :: 120 : 40 =$ C D the conjugate axis.

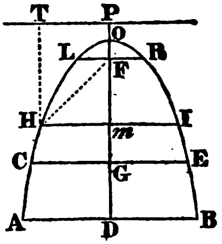
2. If the transverse axis be 80, the greater abscissa 64, and the ordinate 24; what is the length of the conjugate axis? *Ans.* 60.

2. OF THE PARABOLA.

Some general Properties of the Parabola.

1. The distance from the focus to any point in the curve, is equal to the perpendicular distance of that point from the directrix.

Thus, if F be the focus, T P the directrix, and H any point in the curve, then F H = H T.



2. The vertex of the parabola is equidistant from the focus and the directrix.

Viz. F O = O P, where O is the vertex.

3. The distance of the focus from the vertex, is equal to one-fourth of the *latus rectum*.

Viz. F O = $\frac{1}{4}$ of L R.

4. The square of any ordinate to the axis, is equal to the rectangle of the latus rectum, and the abscissa to that ordinate.

Viz. C G^2 = L R × O G.

Hence, if x = any abscissa, y its ordinate, and p the parameter or latus rectum, then $y^2 = p\,x$, any two of which being given, the third is easily found.

5. The abscissas are to each other, as the squares of their ordinates.

Viz. O m : O G :: H m^2 : C G^2.

6. The latus rectum is to the sum of any two ordinates, as the difference of these ordinates is to the difference between their abscissas.

Viz. $LR : CG + Hm :: CG - Hm : OG - Om$.

7. All parabolas are similar to each other, and any two parabolas, having equal parameters and abscissas, are equal to each other.

8. The area of a parabola is equal to $\frac{2}{3}$ of the area of its circumscribing parallelogram.

9. The distance of the focus from the vertex, is equal to the square of any ordinate divided by four times its abscissa.

Viz. $OF = CG^2 \div 4\,OG$.

10. The latus rectum is equal to the square of any ordinate divided by its abscissa.

Viz. $LR = CG^2 \div OG$.

Note. The preceding, and various other properties of the parabola, are demonstrated in *Emerson's* Conic Sections.

PROBLEM XXVI.

To find the Area of a Parabola, its Base and Height being given.*

RULE. Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area.

Example 1. If the base AB, or greatest double ordinate of a parabola AOB, be 53·75, and the abscissa DO, or height of the parabola be 39·25, what is the area?

$(AB \times DO) \times \frac{2}{3} = (53{\cdot}75 \times 39{\cdot}25) \times \frac{2}{3} =$ 1406·4583, the area required.

2. What is the area of a parabola, whose height or abscissa is 10; and the base, or double ordinate, 16? *Ans.* $106\frac{2}{3}$.

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* Any double ordinate AB or CE of a parabola may be called the base, and the axis OD, or any part of it OG, the height; hence the base is any double ordinate, and the height is an abscissa to that double ordinate.

3. Required the area of a parabola, whose base, or double ordinate, is 38, and the height or abscissa, 12? *Ans.* 304.

PROBLEM XXVII.

To find the Area of the Zone of a Parabola, or of the Space included between two parallel double Ordinates.

Rule*. *If the two double ordinates, their distance, and the altitude of the whole parabola be given;* find the area of the whole parabola, and also of the upper segment, the difference will be the area of the zone.

But if the two double ordinates, and their distance only, be given. To the sum of the squares of the two double ordinates, add their product, divide the result by the sum of the two double ordinates, the quotient multiplied by $\frac{2}{3}$ of the altitude of the zone will give the area.

Example 1. In the parabolic zone A B I H, there are given the double ordinate $AB = 10$, the double ordinate $HI = 6$, their distance $Dm = 4$, and the altitude D O of the whole parabola = 6·25, required the area.

---

* The first part of the Rule is self-evident. By the 5th property, p. 78, $DO : mO :: AD^2 : Hm$; now, if $D = AB$ the greater double ordinate, $d = HI$ the less, and $Dm = a$ their distance, then $DO : mO : \frac{1}{4}D^2 : \frac{1}{4}d^2$, or $DO : mO :: D^2 : d^2$, and by division, (Euclid 17 *of* V.) $DO - mO : mO :: D^2 - d^2 : d^2$, viz. $Dm : mO :: D^2 - d^2 : d^2$, or $a : mO :: D^2 - d^2 : d^2$, hence $mO = ad^2 \div (D^2 - d^2)$, consequently $DO = a + \frac{ad^2}{D^2 - d^2}$. Now $D \times \left(a + \frac{ad^2}{D^2 - d^2}\right) \times \frac{2}{3} =$ area of the whole parabola A O B, and $d \times \frac{ad^2}{D^2 - d^2} \times \frac{2}{3} =$ area of the segment H O I, their difference $\frac{D^3 - d^3}{D^2 - d^2} \times \frac{2}{3}a = \frac{D^2 + Dd + d^2}{D + d} \times \frac{2}{3}a$, is the area of the zone A H I B, which is the rule.

$(10 \times 6{\cdot}25) \times \frac{2}{3} = 41\frac{2}{3}$ area of the parabola A O B.
$(6 \times 2{\cdot}25) \times \frac{2}{3} = 9$ area of the segment H O I.

Difference $= 32\frac{2}{3}$ area of the zone A H I B.

When the altitude D O, *of the whole of the parabola, is not given.*

$$\frac{(10^2 + 6^2) + (10 \times 6)}{10 + 6} = 12\tfrac{1}{4}, \text{ and } 12\tfrac{1}{4} \times 4 \times \tfrac{2}{3}$$

$= 32\frac{2}{3}$ area as above.

2. Suppose the double ordinate C E of the zone C E I H to be 25·298, H I = 16, and their distance G *m* = 9, what is the area of the zone? *Ans.* 188·98.

3. Suppose the double ordinate A B = 24, C E = 20, and their distance D G = 5·5, what is the area of the zone A B E C? *Ans.* $121\frac{1}{3}$.

PROBLEM XXVIII.

To find the Length of any Arc of a Parabola, cut off by a double Ordinate.

Rule. 1. To the square of the ordinate add $\frac{4}{3}$ of the square of the abscissa, and the square root of the sum will be half the length of the curve *nearly*.

2. To the square of the ordinate add $\frac{2}{3}$ of the square of the abscissa, and divide four times this sum by the ordinate; subtract the quotient from nine times the half length of the curve found above, and $\frac{2}{5}$ of the remainder will be the whole length of the curve *extremely near* *.

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* This rule is derived from a series which ceases to converge when the abscissa exceeds half the ordinate, or when the double ordinate exceeds the latus rectum; in such cases it ought not to be used. The first part of the rule, which has generally been given, with improper examples as exercises, is derived from the same series.

Example 1. In the parabola H O I, suppose the abscissa m O = 2, and its ordinate H m = 6, what is the length of the curve H O I?

$\sqrt{\text{H}m^2 + \frac{4}{3}\overline{m\text{O}}|^2} = \sqrt{6^2 + (\frac{4}{3} \times 2^2)} = \sqrt{\frac{1}{3}(108 + 16)} = \frac{2}{3}\sqrt{93} = 6{\cdot}429105$, half the length of the curve nearly*.

$(\text{H}m^2 + \frac{2}{3}\,m\text{O}^2) \times \frac{4}{\text{H}m} = 6^2 + (\frac{2}{3} \times 2^2) \times \frac{4}{6} = 25\frac{7}{9}$; then $(6{\cdot}429105 \times 9) - 25\frac{7}{9} = 32{\cdot}084167$, hence $\frac{2}{5} \times 32{\cdot}084167 = 12{\cdot}8336$, the length of the curve H O I required.

2. What is the length of the parabolic curve, whose abscissa is 3, and ordinate 8? *Ans.* 17·384.

PROBLEM XXIX.

In a Parabola, any three of the four following Terms being given, viz. any two Ordinates and their two Abscissas, to find a fourth.

RULE †. 1. Any abscissa is to the square of its ordinate, as any other abscissa is to the square of its ordinate.

2. The square root of any abscissa is to its ordinate, as the square root of any other abscissa is to its ordinate.

Example 1. If the abscissa m O = 9, and the ordinate H m = 6, what is the ordinate A D, when the abscissa D O is 16?

$\therefore\ m\text{O} : \text{H}m^2 :: \text{DO} : \text{AD}^2$, viz. $9 : 36 :: 16 : 64$ the square root of which is 8, the ordinate A D.

$\therefore$ Or, $\sqrt{m\text{O}} : \text{H}m :: \sqrt{\text{DO}} : \text{AD}$, viz. $3 : 6 :: 4 : 8$ the ordinate A D.

2. Required the abscissa corresponding to the ordinate 8, that corresponding to the ordinate 6 being 9. *Ans.* 16.

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* Hence $6{\cdot}429105 \times 2 = 12{\cdot}858210$ the length of the curve nearly.

† See the 5th property, p. 78.

3. If the two abscissas be 4 and 9, and their corresponding ordinates 8 and 12; any three of these being given, it is required to find the fourth.

PROBLEM XXX.

Given two Ordinates perpendicular to the Axis, and their Distance to find the corresponding Abscissas.

Rule. The difference of the squares of the ordinates is to their distance, as the square of either of them is to its corresponding abscissa *.

Example 1. In the parabola AOB, given the ordinate $CG = 5$, the ordinate $Hm = 3$, and their distance $Gm = 4$, to find the two abscissas GO, and mO.

$CG^2 - Hm^2 : Gm :: CG^2 : GO$, viz. $5^2 - 3^2 : 4 :: 5^2 : 6{\cdot}25 = GO$.

$CG^2 - Hm^2 : Gm :: Hm^2 : mO$, viz. $5^2 - 3^2 : 4 :: 3^2 : 2{\cdot}25 = mO$.

Hence the two abscissas are 6·25 and 2·25. *Ans.*

2. If the greater ordinate of a parabola be 10, the less 6, and their distance 6; what are the two abscissas? *Ans.* 9·375 and 3·375.

3. OF THE HYPERBOLA.

Some general Properties of the Hyperbola.

1. The difference between any two straight lines, drawn from the foci to any point in the curve, is equal to the *transverse axis*, or *axis major*.

Viz. $fP - FP = AB$ the axis major.

2. The two foci are equidistant from the centre of the hyperbola.

Viz. $fO = FO$, where O is the centre.

3. The square of the distance of the focus from the centre, is equal to the sum of the squares of the semi-transverse and semi-conjugate.

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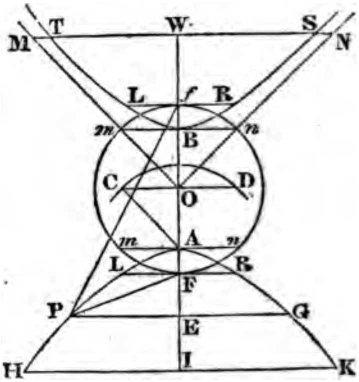
[*] This Rule follows from the demonstration of the rule to Problem XXVII.

Viz. $OF^2 = AO^2 + OC^2$, this is obvious for $OF = AC$ by the construction of the hyperbola, see the definitions, &c. page 8.



4. The rectangle of the distances of the two foci from either vertex, is equal to the square of the semi-conjugate.

Viz. $FA \times fA = OD^2$, or $fB \times FB = OD^2$.

This is very obvious, for $An = OD$ the semi-conjugate, and by EUCLID 35 *of* III, $FA \times fA = An^2$.

5. The rectangle of the distance of either focus from the two vertices, is equal to the square of the semi-conjugate.

Viz. $FA \times FB = An^2 = OD^2$; or, $fB \times fA = OD^2$.

6. The transverse axis is to the conjugate, as the conjugate is to the latus rectum.

Viz. $AB : CD$ (or mn) $:: CD : LR$; hence $LR = CD^2 \div AB$.

7. The square of the transverse axis is to the square of the conjugate, as the rectangle of the distance of the ordinate from each vertex is to the square of the ordinate.

Viz. $AB^2 : CD^2 :: EA \times EB : EP^2$. Hence if $t =$ the transverse axis, $c =$ the conjugate, $x =$ the abscissa EA, and $y =$ the ordinate EP, then will $t + x =$ the other abscissa EB, and $t^2 : c^2 :: x \times (t + x) : y^2$ any three of these quantities being given, the fourth may be found.

8. If a straight line be drawn parallel to the conjugate axis, so as to meet the asymptotes of the curve; the rectangle contained by the segments of this line, between the curve and the asymptotes, is equal to the square of the semi-conjugate axis.

Viz. If OM and ON be the asymptotes to the curve TBS, mBn the conjugate axis, and MWN a straight line parallel to it; then $MS \times SN = Bn^2$.

9. If a straight line be drawn through the curve, meeting the asymptotes, the parts of this line between the curve and the asymptotes, are equal.

Viz. $MT = SN$, and $MS = NT$.

Note. All the foregoing properties are demonstrated in *Emerson's* Conic Sections.

PROBLEM XXXI.

Given the transverse Axis of an Hyperbola, the Conjugate, and the Abscissa, to find the Area.

RULE *. 1. To the product of the transverse and abscissa, add $\frac{5}{7}$ of the square of the abscissa, and multiply the square-root of the sum by 21.

2. Add 4 times the square-root of the product of the transverse and abscissa, to the preceding product, and divide the sum by 75.

3. Divide 4 times the product of the conjugate and abscissa by the transverse; this quotient, multiplied by the former quotient, will give the area of the hyperbola *nearly*.

Example 1. In the hyperbola H A K, if the transverse axis A B = 100, the conjugate C D = 60, and the abscissa I A = 50, what is the area?

First, $\sqrt{(\mathrm{AB} \times \mathrm{IA}) + \frac{5}{7}\,\mathrm{IA}^2} \times 21 = \sqrt{(100 \times 50) + (\frac{5}{7} \times 50^2)} \times 21 = 82{\cdot}375447 \times 21 = 1729{\cdot}884387$.

Second, $4\sqrt{\mathrm{AB} \times \mathrm{IA}} = 4\sqrt{100 \times 50} = 40\sqrt{50} = 282{\cdot}842712$, then $(282{\cdot}842712 + 1729{\cdot}884387) \div 75 = 26{\cdot}83636132$ *quotient*.

Third, $(\mathrm{CD} \times \mathrm{IA} \times 4) \div \mathrm{AB} = (60 \times 50 \times 4) \div 100 = 120$ *quotient*, then $26{\cdot}83636132 \times 120 = 3220{\cdot}3633584$, area of the hyperbola H A K, *nearly*.

2. Required the area of the hyperbola P A G, the abscissa E A being 10, the transverse axis A B = 30, and the conjugate axis C D or *m n* = 18. *Ans.* 151·68.

3. What is the area of the hyperbola T B S, the transverse axis A B being 50, the conjugate axis *m* B *n* 30, and the abscissa B W 25? *Ans.* 805·0909.

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* This Rule is *Sir Isaac Newton's*; it is given in *Robertson's Mensuration*, 2d edition, p. 202; and demonstrated by Dr. *Hutton*, in the quarto edition of his *Mensuration*, p. 376.

PROBLEM XXXII.

To find the Length of any Arc of an Hyperbola, beginning at the Vertex.

RULE[*]. 1. Divide the square of the conjugate axis by the transverse, and the quotient will give the *latus rectum* or parameter[†].

2. To 19 times the transverse axis, add 21 times the parameter, and multiply the sum by the abscissa.

Again. To 9 times the transverse axis, add 21 times the parameter, and multiply the sum by the abscissa.

To each of these products, add 15 times the square of the conjugate axis.

3. Then the less sum is to the greater, as the ordinate is to half the length of the curve.

Example 1. In the hyperbola PAG, the transverse diameter $AB = 80$, the conjugate $CD = 60$, the ordinate $PE = 10$, and the abscissa $EA = 2·1637$, what is the length of the arc ALP?

First. $CD^2 \div AB = 60^2 \div 80 = 45 = LR$ the latus rectum or parameter.

Second. $(19\,AB + 21\,LR) \times EA = (19 \times 80) + (21 \times 45) \times 2·1637 = 5333·5205.$

$(9\,AB + 21\,LR) \times EA = (9 \times 80) + (21 \times 45) \times 2·1637 = 3602·5605.$

Then $5333·5205 + (15 \times 60^2) = 59333·5205.$

And $3602·5605 + (15 \times 60^2) = 57602·5605.$

Third. $57602·5605 : 59333·5205 :: 10 : 10·3005$ the length of the arc ALP, the double of which, viz. 20·601, is the length of the whole curve PLARG.

2. The transverse diameter AB of the hyperbola PAG is 60, the conjugate CD 36, the ordinate PE

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[*] This Rule is deduced from Dr. *Hutton's Mensuration*, quarto edition, p. 364.

[†] By the 6th property, p. 84.

= 24,

= 24, and the abscissa E A = 20, what is the length of the curve P L A R G? *Ans.* 62·652.

3. The transverse axis A B of the hyperbola P A G is 30, the conjugate C D 20, the ordinate P E = 9·797959, and the abscissa E A = 6, what is the length of the curve P A G? *Ans.* 23·388676.

PROBLEM XXXIII.

In an Hyperbola, any three of the four following Terms being given, viz. the transverse and conjugate Diameters, an Ordinate and its Abscissa, to find the fourth *.

CASE I.

To find the ordinate.

Rule. The transverse axis is to the conjugate, as the square-root of the product of the two abscissas is to the ordinate.

Example 1. In the hyperbola H A K, if the transverse axis A B = 100, the conjugate C D = 60, and the abscissa I A = 50, what is the length of the ordinate H I?

$IA + AB = 50 + 100 = 150$ = the greater abscissa I B.

Then $AB : CD :: \sqrt{IA \times IB} : HI$, viz. $100 : 60 :: \sqrt{50 \times 150} : 51{\cdot}961524$, the ordinate H I required.

2. If the transverse axis A B of the hyperbola P A G be 24, the conjugate C D 21, and the abscissa E A = 8, what is the ordinate P E? *Ans.* 14.

3. If the transverse axis A B of the hyperbola T B S be 120, the abscissa W B = 40, the conjugate axis m B n = 72, what is the length of the ordinate T W? *Ans.* 48.

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* See the 7th general property, p. 84.

CASE II.

To find the two abscissas.

Rule. To the square of the conjugate axis, add four times the square of the ordinate, and extract the square-root of the sum.

Then the conjugate is to the transverse, as half the square-root, above found, is to the distance of the ordinate from the centre of the hyperbola.

This distance, added to half the transverse axis, will give the greater abscissa, and their difference will give the less abscissa.

Example 1. If the transverse axis A B of the hyperbola P A G be 24, the conjugate C D 21, and the ordinate P E = 14, what are the two abscissas?

$\sqrt{CD^2 + 4PE^2} = \sqrt{21^2 + (4 \times 14^2)} = \sqrt{1225} = 35$, the half = 17·5, then C D : A B :: 17·5 : E O, viz. 21 : 24 :: 17·5 : 20 = E O.

Now $EO + \frac{1}{2}AB = 20 + 12 = 32$ the abscissa E B.

And $EO - \frac{1}{2}AB = 20 - 12 = 8$ the abscissa E A.

2. If the transverse axis A B of the hyperbola T B S be 120, the conjugate m B n = 72, and the ordinate T W = 48, what are the two abscissas W B and W A? *Ans.* W B = 40, and W A = 160.

3. If the transverse axis A B of the hyperbola P A G be 30, the conjugate C D or m A n = 18, the ordinate P E = 12, what are the two abscissas E A and E B? *Ans.* E A = 10, E B = 40.

CASE III.

To find the conjugate axis.

Rule. The square-root of the product of the two abscissas, is to the ordinate as the transverse axis is to the conjugate axis.

Example 1. The transverse axis A B of the hyperbola H A K is 100, the ordinate H I = 51·961524, and the abscissa I B = 150, what is the conjugate C D or *m* A *n*?

I B — A B = 150 — 100 = 50 the abscissa I A; $\sqrt{\text{IB} \times \text{IA}}$: H I :: A B : C D, viz. $\sqrt{150 \times 50}$: 51·961524 :: 100 : 60, the conjugate axis C D or *m* A *n*.

2. If the transverse axis A B of the hyperbola P A G be 60, the ordinate P E = 24, and the abscissa E A = 20, what is the conjugate axis C D? *Ans.* 36.

3. If the transverse axis A B of the hyperbola T B S be 120, the ordinate T W = 48, and the abscissa W B = 40, what is the conjugate axis *m* B *n*? *Ans.* 72.

CASE IV.

To find the transverse axis.

RULE. To the square of the conjugate axis, add four times the square of the ordinate, and extract the square-root of the sum; to which sum add the conjugate, if the less abscissa be used; or subtract it, if the greater abscissa be used, and take half the result.

Then the square of the ordinate is to the half result found above, as the product of the conjugate and abscissa is to the transverse.

Example 1. In the hyperbola P A G, the less abscissa E A = 8, its ordinate P E = 14, what is the transverse A B, if the conjugate *m* A *n* or C D be 21?

$\sqrt{\text{CD}^2 + 4\,\text{PE}^2} + \text{CD} = \sqrt{21^2 + (4 \times 14^2)} + 21 = 35 + 21 = 56$, the half of which is 28.

Then $14^2 : 28 :: 21 \times 8 : 24$, the transverse axis A B.

2. In the hyperbola P A G, the greater abscissa E B = 40, the ordinate P E = 12, the conjugate axis C D = 18, what is the length of the transverse axis A B? *Ans.* 30.

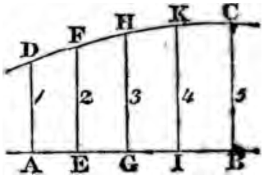
3. In the hyperbola T B S, the less abscissa W B = 40, the ordinate T W = 48, and the conjugate axis *m* B *n* = 72, what is the transverse? *Ans.* 120.

PROBLEM XXXIV.

To find the Area of a Curve by equidistant Ordinates, the Base being divided into any even Number of equal Parts.

RULE [*]. To the sum of the first and last ordinates, add 4 times the sum of all the even ones, and twice the sum of all the odd ones, rejecting the first and last: $\frac{1}{3}$ of the result, multiplied by the common distance of the ordinates, will give the area, nearly.

Example 1. Required the area of the curved space A B C D, the five equidistant ordinates being A D = 10, E F = 11, G H = 14, I K = 16, and B C = 16, the whole base A B = 20.



A D + B C + 4 × (E F + I K) + (2 × G H) = 10 + 16 + 4 × (11 + 16) + (2 × 14) = 26 + 108 + 28 = 162, and $\frac{162}{3} \times 5$ = 270 the area required [†].

2. Suppose the quadrant of a circle, whose radius is 1, to be divided into 11 equidistant ordinates, the first of which is 0, the eleventh = 1, the common difference between the ordinates $\frac{1}{10}$ [‡], the second

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[*] See *Moss's Gauging*, Dr. *Hutton's Mensuration*, quarto edition, or *Emerson's Differential Method*, p. 42, &c.

[†] The area may be found by Problem VIII. p. 37, supposing each of the figures A E F D, E G H F, &c. as a trapezoid.

[‡] $\sqrt{\frac{1}{10} \times 1\frac{9}{10}} = \frac{1}{10}\sqrt{19} = \cdot4358899$ the second ordinate, $\sqrt{\frac{2}{10} \times 1\frac{8}{10}} = \frac{1}{10}\sqrt{36} = \cdot6$ the third ordinate, &c. by the property of the circle.

ordinate ·4358899, third ·6, fourth ·7141428, fifth ·8, sixth ·8660254, seventh ·9165151, eighth ·9539392, ninth ·9797959, and the tenth ·9949874, required the area. *Ans.* ·781752.

3. Given the lengths of 9 equidistant ordinates of an irregular figure, viz. 14, 15, 16, 17, 18, 20, 22, 23, 25 feet, and the common distance 2 feet, required the area? *Ans.* $300\frac{2}{3}$ feet.

4. Given the lengths of three equidistant ordinates, viz. A D = 5, E F = 7, and G H = 8, also the length of the base A G = 10, to find the area of the curve A G H D? *Ans.* $68\frac{1}{3}$.

5. Given eleven ordinates to an hyperbola, between the asymptote and the curve, viz. $\frac{10}{10}$, $\frac{10}{11}$, $\frac{10}{12}$, $\frac{10}{13}$, $\frac{10}{14}$, $\frac{10}{15}$, $\frac{10}{16}$, $\frac{10}{17}$, $\frac{10}{18}$, $\frac{10}{19}$, $\frac{10}{20}$; the whole length of the base of the figure being 1, and the common distance of the ordinates $\frac{1}{10}$; to find the area of the curve.

Ans. ·693150218 being nearly equal to the hyperbolic logarithm of 2.

6. The abscissa of a parabola is 2, and the base or double ordinate 12, what is the area?

Ans. Here the middle ordinate, being the abscissa, is 2, the first and last ordinates are each = 0, and the common distance = 6; hence the area = 16.

THE

COMPLETE MEASURER.

MENSURATION OF SOLIDS.

DEFINITIONS.

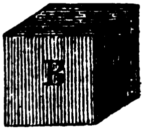
1. SOLID bodies are such as consist of length, breadth, and thickness; as stone, timber, globes, &c.

A TABLE OF CUBIC-MEASURE.

1728	cubic or solid inches, make	1	solid foot.
27	feet	1	yard.
166⅜	yards	1	pole.
64000	poles	1	furlong.
512	furlongs	1	mile.

2. The least solid measure is a cubic inch, and all solids are measured by cubes, whose sides are inches, feet, yards, &c.; and the solidity of a body is said to be so many cubic inches, feet, yards, &c. as will fill the same space as the solid, or as the solid will contain.

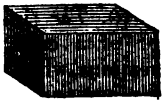
3. *A cube* is a solid, comprehended by six equal square surfaces, as B.



4. *A Prism* is a solid figure, the ends whereof are parallel, equal, and similar plane figures, and the sides which connect the ends are parallelograms.

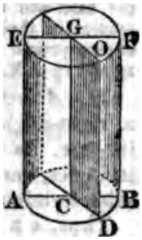
Prisms take particular names from the figure of their bases, or ends, viz. triangular, square, rectangular, pentagonal, hexagonal, &c.

5. *A*



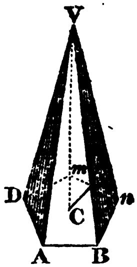
5. *A parallelopipedon* is a prism bounded by six parallelograms, every opposite two of which are equal, similar, and parallel.

If the bounding parallelograms be right-angled, and perpendicular to each other, it is called a rectangular parallelopipedon.



6. *A cylinder* is formed by the revolution of a right-angled parallelogram about one of its sides, which remains fixed, and its bases are two equal and parallel circles.

Thus, the right-angled parallelogram G C D O, by revolving about the fixed axis G C, will describe the cylinder A B F E, and the two ends, C D and G O, will describe the circular bases A D B and E O F.



7. *A pyramid** is a solid, having any rectilineal plane figure for its base, and its sides are plane triangles, having all their vertices meeting together in a point above the plane of the base, which point of meeting is called the vertex of the pyramid.

8. *The altitude* of a pyramid is a straight line drawn from its vertex to the middle of the base.

Thus C V is the altitude of the pyramid A B *n* V D.

9. *The slant height* of a pyramid is a straight line drawn from the vertex to the middle of one of the sides of the base.

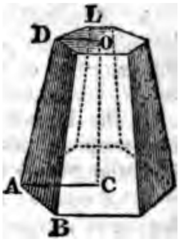
Thus V O, drawn from the vertex V, to the point O, the middle of the side *m n* of the base, is the slant height.

---

* A cone, already defined at the beginning of the article Conic Sections, may be considered as a pyramid, having a regular polygon of an infinite number of sides for its base.

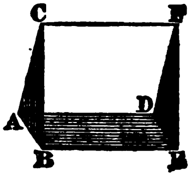
10. *The frustum* of a pyramid, or of a cone, is that part which remains, when the top is cut off by a plane parallel to the base.

Note. If a pyramid, or a cone, be cut by a plane parallel to its base, the section will be similar to the base; and the section and the base will be to each other, as the squares of their distances from the vertex of the pyramid or cone. (*Keith's Geometry*, 12 and 13 of X.)

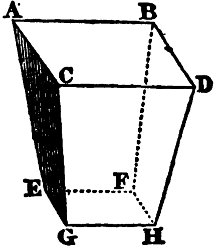


11. *A wedge* is a solid, having a right-angled parallelogram for its base, its two ends being plane triangles, and its two opposite sides terminating in an edge.

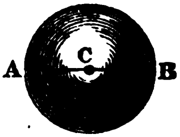
Thus, A B E D is the base, A B C and D E F the two ends, and C F the edge.



12. *A prismoid* is a solid, having for its two ends two right-angled parallelograms, parallel to each other, and its upright sides are four trapezoids.



13. *A sphere*, or globe, is a solid, generated by the revolution of a semicircircle about its diameter, which remains fixed.



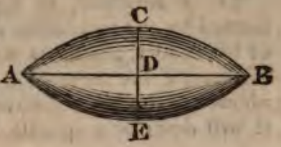
14. *The centre* of a sphere is a point within the figure, every where equally distant from the convex surface.

15. *The diameter* or axis of a sphere, is a straight line passing through the centre, and terminated at each extremity by the surface of the sphere.

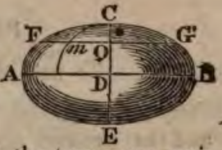
16. *The zone* of a sphere is that part which is contained between two parallel planes; and if these planes be equally distant from the centre, it is called the *middle* zone of a sphere.

17. *The segment* of a sphere is any part of it cut off by a plane. If the plane pass through the centre, it will divide the sphere into two equal parts, or *hemispheres.*

18. *A circular spindle* is a solid, generated by the revolution of a segment of a circle about its chord, which remains fixed.



19. *A spheroid* is a solid, generated by the revolution of a semi-ellipsis about one of its axis, which remains fixed.



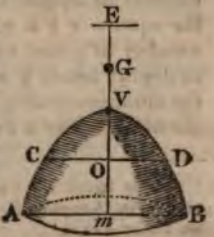
If the ellipsis revolve round the transverse axis A B, the figure is called a *prolate,* or oblong spheroid; if the ellipsis revolve round the shorter axis C E, the figure is called an *oblate* spheroid.

20. *A segment* of a spheroid is a part cut off by a plane, parallel to the longer or shorter axis.

21. *An elliptic spindle* is a solid, generated by the revolution of a segment of an ellipsis about its base, or chord.

22. *A parabolic conoid,* or paraboloid, is a solid formed by the revolution of a semi-parabola about its axis.

Thus, the parabolic conoid A C V D B, is formed by the revolution of the semi-parabola A C V, about its axis V *m.*



23. *An hyperbolic conoid**, or hyperboloid, is a solid formed by the revolution of a semi-hyperbola about its greater abscissa, or transverse axis produced.

Thus, the hyperbolic conoid A C V D B, is formed by the revolution of the semi-hyperbola A C V, about its greater abscissa *m* E, or the transverse axis E V produced.

24. *A parabolic*, or *hyperbolic spindle*, is a solid formed by the revolution of a segment of a parabola, or hyperbola, about its double ordinate.

Thus, if the segment C V D be supposed to resolve about the double ordinate C D, which remains fixed, it will describe a spindle.

PROBLEM I.

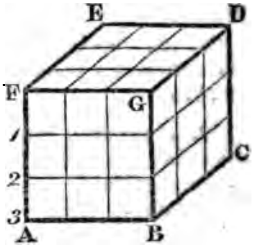
To find the Solidity of a Cube.

Rule. Multiply the side of the cube by itself, and that product again by the side, and the last product will be the solidity required.

Note. The surface of a cube is six times the square of its side.

The side of a cube may be found by extracting the cube-root of its solidity.

Example 1. If the side of a cube be 3 inches, what is the solidity?



Here, 3 × 3 = 9 the number of square inches in the square E F G D, or the number of cubes of 1 inch in depth; hence 9 × 3 = 27 the number of cubic inches in the whole figure A B C D E F G.

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* The figure A C V D B is an hyperbola; and as it differs so very little, in appearance, from the parabola, both the definitions are referred to it.

2. What is the solidity of a cubical piece of stone, each side being 17·5 inches? *Ans.* 3·10149 feet.

3. What is the solidity of a cube, whose side is 19 feet 4 inches.

Ans. 7226 feet, 4 inches, 5 parts, 4 seconds.

4. A cellar is to be dug, whose length, breadth, and depth, are each 10 feet 4 inches: how many solid feet does it contain, and what will it cost digging, at 1 shilling per solid yard?

$$\textit{Ans.} \begin{cases} \text{The content is } 1103\tfrac{10}{27} \text{ feet.} \\ \text{Expense £}20 : 0 : 10\tfrac{94}{243}. \end{cases}$$

5. How many three inch cubes may be cut out of a 12 inch cube? *Ans.* 64.

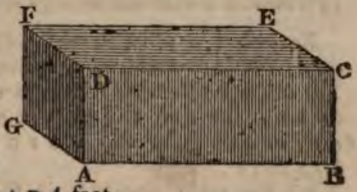
PROBLEM II.

To find the Solidity of a Parallelopipedon.

Rule. Multiply the length by the breadth, and that product by the depth, or altitude, and the last product will be the solidity.

Note. The surface of a parallelopipedon is equal to the sum of the areas of its sides and ends.

Example 1. Required the solidity of the parallelopipedon ABCDEFG, the length AB being 12 feet, the breadth AG 3·5 feet, and thickness AD 4 feet.



AB × AG × AD = 12 × 3·5 × 4 = 168 feet the solidity of the parallelopipedon.

2. If a piece of timber be 25 inches broad, 9 inches deep, and 25 feet long, how many solid feet are contained therein? *Ans.* 39 feet 0 inches 9 parts.

3. If a piece of timber be 15 inches square at the end, and 18 feet long, how many solid feet are con-

tained in it; and suppose I want to cut off a solid foot from one end of it, at what distance from the end must I cut it?

Ans. The solidity is 28·125 feet.
And 7·68 inches in length will make 1 foot.

4. If a piece of squared timber be 2 feet 9 inches deep, 1 foot 7 inches broad, and 16 feet 9 inches long, how many feet of timber are in that piece; and how much in length will make a solid foot?

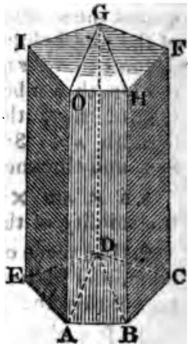
Ans. The solid content is 72·93 feet, or 72 feet, 11 inches 2 parts 3 seconds;
and 2·756 inc. in length make 1 solid foot.

PROBLEM III.

To find the Solidity of a Prism.

Rule. Multiply the area of the base, by the perpendicular height of the prism, and the product will be the solidity.

Note. The surface of a prism is equal to the sum of the areas of its two ends, and each of its sides. Or, the circumference of the end of the solid, multiplied by the length, will give the upright surface, to which add the areas of the two ends.



Example 1. What is the solidity of a pentagonal prism A B C F G I E, whose length G D or I E is 15 feet, and each side O H, of its regular end O H F G I 3·75 feet?

By the table page 41, if the side of a regular pentagon be 1, the area will be 1·720477, hence $1·720477 \times 3·75^2 = 24·1942$ square feet the area of the end O H F G I; which, multiplied by 15 feet, the length, gives 362·913 solid feet, the content of the whole figure.

2. Each side of the base of a triangular prism is 17 inches, and the length 12 feet 5 inches; what are the solidity and surface?

Ans. $\begin{cases}\text{The solidity 10·79 feet.}\\ \text{Surface 54·5089 feet.}\end{cases}$

3. A hexagonal prism measures 28 inches across the centre of the end, from corner to corner, and is 134 inches in length; required the solidity and surface.

Ans. $\begin{cases}\text{The solidity 39·48835 feet.}\\ \text{Surface 85·2392 feet.}\end{cases}$

4. A decagonal prism, or pillar, measures 50 inches in circumference, and is 30 feet high; required the solidity and surface.

Ans. $\begin{cases}\text{The solidity 40·074 feet.}\\ \text{Surface 127·6716 feet.}\end{cases}$

5. The gallery of a church is supported by 10 octagonal prisms of wood, which measure each 48 inches in circumference, and are each 12 feet high; what will be the expense of painting them at 10 pence per square yard? *Ans.* £2. : 7s. : $9\frac{1}{3}$.

6. A trapezoidal prism of earth, or part of a canal, is to be dug, whose perpendicular depth is 10 yards, the width at the top 20 yards, at the bottom 16, and the length 50 yards, the two ends being cut perpendicularly down; how many solid yards of earth are contained in this part? *Ans.* 9000 yards.

PROBLEM IV.

To find the Solidity of a Cylinder.*

RULE. Multiply the area of the base, by the perpendicular height of the cylinder, and the product will be the solidity.

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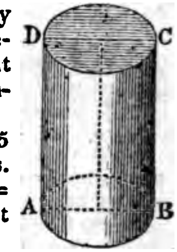
* Rules might here be inserted for finding the solidity and superficies of the *ungulas* or *hoofs* of a cylinder, viz. sections formed by cutting a cylinder parallel or oblique to the axis, of which

Note. To find the surface of a cylinder, multiply the circumference of the base by the height of the cylinder, and the product will be the upright, or convex, surface; to which add twice the area of the base for the whole surface.



Example 1. What is the solidity of a cylinder A B C D, whose diameter A B is 21·5 inches, and the height B C 16 feet? The superficial content is also required.

First, $21{\cdot}5^2 \times {\cdot}7854 = 353{\cdot}05115$ area of the base in square inches. Then $(363{\cdot}05115 \div 144) \times 16 = 363{\cdot}05115 \div 9 = 40{\cdot}339$ cubic feet the solidity required.

To find the superficies.

$21{\cdot}5 \times 3{\cdot}1416 = 67{\cdot}5444$ the circumference of the base, then $67{\cdot}5444 \times 16 \times 12 = 12968{\cdot}5248$ square inches, the convex surface; and $21{\cdot}5^2 \times {\cdot}7854 \times 2 = 726{\cdot}1023$ square inches, double the area of the base; lastly, $(12968{\cdot}5248 + 726{\cdot}1023) \div 144 = 13694{\cdot}6271 \div 144 = 95{\cdot}1$ feet the whole superficies.

2. If a piece of timber be 96 inches in circumference, and 18 feet long, how many feet of timber are contained in it, supposing it to be perfectly cylindrical? *Ans.* 91·676 feet.

3. If a piece of timber, perfectly cylindrical, be 36 inches in circumference, and 20 feet long, how many solid feet are contained therein? *Ans.* 81·74634 feet.

---

which there are *four* varieties. 1. When the section is parallel to the axis. 2. When the section is oblique to the axis, but does not meet the base. 3. When the section passes through the base of the cylinder, and cuts one of its sides. 4. When the section passes obliquely through both ends of the cylinder. But as these sections are more curious than useful, they are here omitted. The reader will find them fully treated of in the large edition of Dr. *Hutton's Mensuration*.

4. The diameter of the base of a cylinder is 20·75 inches, and its length 4 feet 7 inches; what are its solidity and surface?

Ans. $\begin{cases} \text{Solidity } 10{\cdot}7633 \text{ feet.} \\ \text{Surface } 29{\cdot}595 \text{ feet.} \end{cases}$

5. I have a rolling stone 44 inches in circumference, and am to cut off 3 cubic feet from one end; where must the section be made?

Ans. at 33·64 inches.

6. A person bespoke an iron roller for a garden, the outside diameter to be 20 inches, the thickness of the metal 1½ inch, and the length of the roller 50 inches; now supposing every cubic inch to weigh 4¼ ounces, what will the whole come to at 3¾d. per pound?

Ans. $\begin{cases} \text{Solidity } 4358{\cdot}97 \text{ inches.} \\ \text{Weight } 1157{\cdot}8514 \text{ pounds.} \\ \text{Cost £15. : 13s. : 7d.} \end{cases}$

PROBLEM V.

To find the Solidity of a Pyramid.

Rule. Multiply the area of the base by one-third part of the height, and the product will be the solidity *.

Note. To find the surface of a pyramid, multiply the slant height by half the circumference of the base, and the product will be the surface of the sides; to which add the area of the base for the whole surface.

The *slant height* of a pyramid is the perpendicular distance from the vertex to the middle of one of the sides; and the perpendicular height is a straight line drawn from the vertex to the middle or centre of the base.

In order to find the slant height of a pyramid from the perpendicular height being given, or the perpendicular height from the slant height being given, it will be necessary to find the distance

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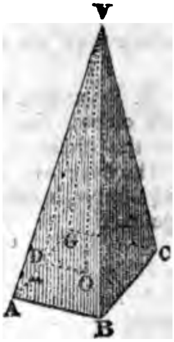
* Every pyramid is the third of a prism, having the same base and height. Euclid 7 of XII cor. or *Keith's Geom.* 16 of X, corol.

between the centre of the base of the pyramid, and the middle of one of its sides; thus, multiply the tabular perpendicular (*Problem* X, p. 41), corresponding to the polygonal base, by one of the sides of the base, and the product will give the distance of the middle of that side from the centre. The square of this distance, added to the square of the perpendicular height, will give the square of the slant height.

Example 1. Let A B C V be a square pyramid, each side of the base A B being 18·5 inches, and the perpendicular height O V = 15 feet, required the solidity and superficies.

18·5 × 18·5 = 342·25 area of the base A B C G, in inches; hence (342·25 ÷ 144) × (15 ÷ 3) = 11·88⅓ feet, the solidity.



To find the superficies.

$\sqrt{OD^2 + OV^2} = DV$ the slant height $= \sqrt{9{\cdot}25^2 + 180^2} = 180{\cdot}24$ nearly.

18·5 × 2 = 37 half the circumference of the base, which multiplied by 180·24, gives 6668·88 for the surface of the sides in inches; hence (6668·88 + 342·25) ÷ 144 = 48·68 feet the whole superficies.

2. Each side of the base of a triangular pyramid is 21·5 inches, and the perpendicular height 16 feet, what are the solidity and surface?

Ans. { Solidity 7·41477 feet.
Superficies 44·41272 feet.

3. Each side of the base of a heptagonal pyramid is 15 inches, and the perpendicular height 13·5 feet, what are the solidity and surface?

Ans. { Solidity 25·55 feet.
Surface 65·0128 feet.

4. There is a triangular pyramid, the sides of its base are 13, 14, and 15 feet, and its altitude 63 feet, what is the solidity? *Ans.* 1764 feet.

5. Each side of the base of a hexagonal pyramid is 10, and the perpendicular height 45, what are the solidity and superficies?

Ans. { Solidity 3897·1143 feet.
Superficies 1634·580328 feet.

6. Required the solidity and surface of a square pyramid, each side of the base being 3 feet, and the perpendicular height 24 feet.

Ans. { Solidity 72 feet.
Surface 153·2808 feet.

7. A square pyramidal stone, whose slant height is 21 feet and each side of its square base 30 inches, is to be sold at 7s. per solid foot, and the polishing of the upright surface will cost 8d. per foot; what will be the expense of the stone when finished?

Ans. £18. : 15s. : 8¼d.

8. The spire of a church is an octagonal pyramid (built of stone) each side at the base being 5 feet 10 inches, and its perpendicular height 45 feet; also each side of the cavity, or hollow part, at the base is 4 feet 11 inches, and its perpendicular height 41 feet; I desire to know how many solid yards of stone the spire contains?

Ans. { Solidity of the whole 2464·509614 feet.
The cavity 1595·180393 feet.
And the stone-work 32·19738 yards.

PROBLEM VI.

To find the Solidity of a Cone.

RULE. Multiply the area of the base by one-third part of the perpendicular height, and the product will be the solidity*.

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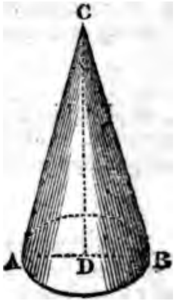
* Every cone is the third part of a cylinder, having the same base and height. EUCLID, 10 of XII. or *Keith's Geom.* 17 of X.

Note. To find the surface of a cone, multiply the circumference of the base of the cone by the slant height, and half the product will be the convex surface, to which add the area of the base, and the sum will be the whole surface of the cone.



Example 1. The diameter A B of the base of the cone A B C is 26·5 inches, and the perpendicular height C D = 16·5 feet, required the solidity and superficies of the cone.

$26{\cdot}5^2 \times {\cdot}7854 = 551{\cdot}54715$ area of the base in inches.

$(551{\cdot}54715 \times 5{\cdot}5) \div 144 = 21{\cdot}066$ feet.

To find the Superficies.

$\sqrt{AD^2 + DC^2} = AC$ the slant height $= \sqrt{13{\cdot}25^2 + 198^2} = 198{\cdot}4428$; then $26{\cdot}5 \times 3{\cdot}1416 = 83{\cdot}2524$ inches, circumference of the base. Again, $(83{\cdot}2524 \times 198{\cdot}4428) \div 2 = 8260{\cdot}41968136$ inches the convex surface. Lastly, $(8260{\cdot}41968136 + 551{\cdot}54715) \div 144 = 61{\cdot}194$ feet, the whole surface.

2. The diameter of the base of a cone is 10, and its perpendicular height 68·1; required the solidity and superficies.

Ans. { Solidity 1782·858.
Superficies 1151·134076.

3. What is the solidity of an elliptical cone, the greater diameter of its base being 15·2, the less 10, and the perpendicular height 22? *Ans.* 875·4592.

4. What are the solidity and superficies of a cone, whose perpendicular height is 10·5 feet, and the circumference of its base 9 feet?

Ans. { Solidity 22·56093 feet.
Superficies 54·1336 feet.

5. What are the solidity and superficies of a cone, whose slant height is 32 feet, and the circumference of its base 24 feet?

Ans. { Solidity 485·4433316 feet.
Surface 429·83804 feet.

6. What will the painting of a conical church spire come to at 8d. per yard; supposing the circumference of the base to be 64 feet, and the perpendicular height 118 feet?

Ans. { Superficies 3789·76 feet.
Expense £14. : 0s. : 8¾d.

PROBLEM VII.

*To find the Solidity of the Frustum of a Pyramid**.

GENERAL RULE.

Multiply the areas of the two ends together, and to the square root of the product add the two areas; that

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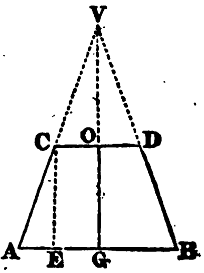
* Suppose A B V to be a pyramid cut by a plane C D, parallel to its base.



Let D = A B, a line, or diameter of the greater base.

Let d = C D, a similar line of the less base.

b = G O, the height of the frustum.

A = the area of the greater base.

a = the area of the less base.

Then A G — C O : G O :: C O : O V, by similar triangles.

Viz. $\frac{1}{2}D - \frac{1}{2}d : b :: \frac{1}{2}d : (bd \div D - d) = OV$.

$$\text{But } GO + OV = b + \frac{bd}{D-d} = \frac{bD}{D-d} = GV.$$

$$\frac{bd}{D-d} \times \frac{a}{3} = \text{solidity of the pyramid } CDV.$$

$$\frac{bD}{D-d} \times \frac{A}{3} = \text{solidity of the pyramid } ABV.$$

that sum, multiplied by one third of the height, gives the solidity of any frustum.

Hence $\left(\frac{b\,D}{D - d} \times \frac{A}{3}\right) - \left(\frac{b\,d}{D - d} \times \frac{a}{3}\right) = \frac{A\,D - a\,d}{D - d} \times \frac{b}{3}$ the solidity of the frustum A B D C. Now all similar plane figures are to each other as the squares of their homologous sides. Therefore $A : a :: D^2 : d^2$ or $\frac{A}{D^2} = \frac{a}{d^2}$, put each of these equal to m; then $A = m\,D^2$ and $a = m\,d^2$, and multiplying these quantities together, we get $a\,A = m^2\,D^2\,d^2$, consequently $\sqrt{a\,A} = m\,D\,d$. For A and a substitute their values in the expression for the frustum's solidity, and it becomes $\frac{m\,D^3 - m\,d^3}{D - d} \times \frac{b}{3} = \frac{D^3 - d^3}{D - d} \times \frac{b}{3} \times m = (D^2 + D\,d + d^2) \times \frac{b}{3} \times m = m\,D^2 + m\,D\,d + m\,d^2 \times \frac{b}{3}$: restore the values of A and a, &c. and the rule becomes $(A + \sqrt{a\,A} + a) \times \frac{b}{3}$, which is the *general Rule.*

Corollary 1. If the bases be squares, of which D and d are each a side, then $\overline{D^2 + D\,d + d^2}$; $\times \frac{b}{3}$ is the solidity, and is the second rule.

Cor. 2. If the bases be circles, of which D and d are diameters, then $\overline{D^2 + D\,d + d^2}$; $\times b \times \cdot 2618$ is the solidity.

Cor. 3. If the bases be regular polygons, and t represent the tabular number in Problem X, p. 41, also D and d represent a side of each base, the rule becomes $(\overline{D^2 \times t} + \sqrt{D^2\,t \times d^2\,t} + d^2\,t)$; $\times \frac{b}{3} = (D^2 + D\,d + d^2) \times \frac{t}{3} \times b$, the solidity, which is Rule III.

That the rule for finding the superficies of a frustum is true, will readily appear, when we consider that if the ends be regular polygons, the sides will consist of as many trapezoids as there are sides in the polygon; and the common height of these trapezoids will be the slant height of the frustum; and the sum of their parallel sides, the sum of the perimeters of the ends.

RULE II.

If the Ends be Squares.

To the rectangle (or product) of the sides of the two ends, add the sum of their squares; that sum, being multiplied into one-third of the frustum's height, will give its solidity.

RULE III.

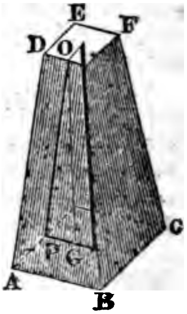
If the Ends be regular Polygons.

Add the square of a side of each end of the frustum, and the product of those sides into one sum; multiply this sum by one-third of the tabular area belonging to the polygon (*Problem* X, p. 41), and this product by the height, for the solidity.

Note. To find the superficies, multiply half the sum of the perimeters of the two ends by the slant height, and the product will be the surface of the sides; to which add the areas of the ends, and the sum will be the whole surface of the frustum.

The slant height of any frustum, whose ends are regular polygons, is a straight line drawn from the middle of one side of the less end, to the middle of its parallel side at the greater end. And the perpendicular height is a straight line drawn from the centre of the less end, to the centre of the greater. Hence the slant height, and perpendicular height, are two sides of a right-angled triangle; the base of which is equal to the difference between the radii of the inscribed circles of the two ends of the frustum. This base may always be found, by multiplying the difference between a side at each end of the frustum, by the tabular perpendicular Problem X, p. 41.

Example 1. What are the solidity and superficies of the frustum of a square pyramid, each side of the greater base A B being 18 inches, each side of the less base D E = 12 inches, and the perpendicular height O G 18 feet?



By the general Rule.

$18 \times 18 = 324$ inches, area of the end A B C.
$12 \times 12 = 144$ inches, area of the end D E F.
$\sqrt{324 \times 144} = 216$ inches, the mean area.

Sum = 684 inches = 4·75 feet.

Then $4{\cdot}75 \times (18 \div 3) = 28{\cdot}5$ feet the solidity.

By Rule II.

$(18 \times 12) + 18^2 + 12^2 = 684$ inches = 4·75 feet.

Then $4{\cdot}75 \times (18 \div 3) = 28{\cdot}5$ feet, the solidity as before.

To find the Superficies.

$18 \times 4 = 72$, the perimeter of the greater end.
$12 \times 4 = 48$, the perimeter of the less end.

Sum = 120, the half of which is 60.

$(18 - 12) \times {\cdot}5$, the tabular perpendicular $= 3 =$ P G, then $\sqrt{PG^2 + GO^2} = PO$, the slant height $= \sqrt{3^2 \times 216^2} = 216{\cdot}02083$, and $216{\cdot}02083 \times 60 = 12961{\cdot}2498$ inches; to this product add the areas of the two ends 324 and 144, and the whole surface will be 13429·2498 inches, or 93·2586 feet.

2. What are the solidity and superficies of the frustum of a triangular pyramid, each side of the greater base being 25 inches, each side of the less base 9 inches, and the length 15 feet?

Ans. { Solidity 13·9977 feet.
Surface 65·893933 feet.

3. What are the solidity and surface of the frustum of an octagonal pyramid, each side of the greater base being 9 inches, each side of the less base 5 inches, and the length 10·5 feet?

Ans. { Solidity 17·721 feet.
Surface 52·590223 feet.

4. If each side of the greater end of a piece of squared timber be 25 inches, each side of the less end 9 inches, and the length 20 feet; how many solid feet are contained in it?

Ans. by Rule 2, 43·1018 feet.

5. If a piece of timber be 32 inches broad, and 20 inches deep, at the greater end; and 10 inches broad, and 6 inches deep, at the less end; how many solid feet are contained therein, the length being 18 feet? *Ans.* by the general Rule, 37·3316 feet.

6. A portico is supported by four pillars of marble, each having eight equal sides, whose breadth at the greater end is 7·5 inches, and at the less end 4½ inches, and their length 12 feet 9 inches; I desire to know how many solid feet they contain, and what they will come to at 12s. 10d. per solid foot?

Ans. { Solidity 60·52927823 feet.
Expense £38. : 16s. : 9½d.

7. In a frustum of a square pyramid, each side of the greater end is 5 feet, each side of the less end 3 feet, and the height 8 feet; required the height and solidity of that pyramid of which this frustum is a part.

Ans. { The height is 20 feet, see the note, p. 105.
The solidity is 166⅔ feet.

8. If each side of the greater base of the frustum of a hexagonal pyramid be 13, each side of the less base 8, and the length 24, what are the solidity and superficies?

Ans. { Solidity 7004·412896.
Superficies 2141·7642.

PROBLEM VIII.

To find the Solidity of the Frustum of a Cone[*].

RULE I. Multiply the areas of the two ends together, and to the square-root of the product add the two areas; that sum, multiplied by one-third of the height, will be the solidity of the frustum.

RULE II. To the product of the diameters of the two ends, add the sum of their squares; this sum, multiplied by the height, and then by ·2618, or one third of ·7854, the last product will be the solidity.

Note: To find the surface of the frustum of a cone, multiply half the sum of the perimeters of the two ends by the slant height, and the product will be the convex surface; to which add the areas of the two ends, and the sum will be the whole surface of the frustum.

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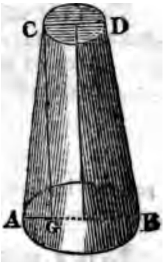
[*] See the Note to Problem VII. page 106.

Rules might here be inserted, were they of any material use, for finding the solidity of the *ungulas* or *hoofs* of the frustum of a cone, of which there are four varieties. 1. When the axis of the section passes through the opposite extremities of two parallel diameters of the ends of the frustum. 2. When the axis of the section, drawn from one extremity of a diameter of the less end, cuts the parallel diameter of the greater end, so as to make a less angle with that diameter, than the opposite side of the frustum does. 3. When the axis of the section, drawn from one extremity of a diameter of the less end, to cut the parallel diameter of the greater end, is parallel to the opposite side of the frustum. 4. When the axis of the section, drawn from one extremity of a diameter of the less end, cuts the diameter of the greater end, so as to make a greater angle with that diameter, than the opposite side of the frustum does. For information on these subjects, the learner is referred to the large edition of Dr. *Hutton's Mensuration.*

Example 1. What are the solidity and superficies of the frustum of a cone, whose greater diameter A B is 18 inches, less diameter C D 9 inches, and the length C G 14·25 feet?



By Rule I.

$18^2 \times {\cdot}7854 = 254{\cdot}4696$ inches, area of the greater end.

$9^2 \times {\cdot}7854 = 63{\cdot}6174$ inches, area of the less end.

$\sqrt{254{\cdot}4696 \times 63{\cdot}6174} = 127{\cdot}2348$ inches, the mean area.

Sum $= 445{\cdot}3218$ inches $=$ 3·0925125 feet.

Then $3{\cdot}0925125 \times (14{\cdot}25 \div 3) = 14{\cdot}68943$ feet, the solidity.

By Rule II.

$(18 \times 9) + 18^2 + 9^2 = 567$ square inches $=$ 3·9375 feet; then, $3{\cdot}9375 \times 14{\cdot}25 \times {\cdot}2618 = 14{\cdot}68943$ feet, the solidity.

To find the Superficies.

$\sqrt{AG^2 + GC^2} = AC$ the slant height $= \sqrt{4{\cdot}5^2 + 171^2} = 171{\cdot}0592$.

$18 \times 3{\cdot}1416 = 56{\cdot}5488$ circumference of the greater end.

$9 \times 3{\cdot}1416 = 28{\cdot}2744$ circumference of the less end.

Sum $= 84{\cdot}8232$, the half of which is 42·4116.

Then 171·0592 × 42·4116 = 7254·89436672, the convex surface.

Area of the greater end = 254·4696
Area of the less end = 63·6174

Sum = 7572·98136672

square inches = 52·59 square feet, the whole superficies.

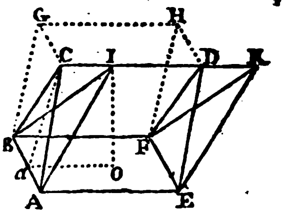
2. If a piece of timber be 9 inches in diameter at the less end, 36 inches at the greater end, and 24 feet long; how many solid feet are contained therein? *Ans.* 74·2203 feet.

3. If a piece of timber be 136 inches in circumference at one end, 32 inches at the other, and 21 feet long; how many solid feet are contained therein? *Ans.* 92·34816 feet.

PROBLEM IX.

To find the Solidity of a Wedge.

RULE*. To twice the length of the base, add the length of the edge; multiply the sum by the breadth.



* *Demonstration.* Let ABCDEF represent a wedge: now, when the length of the edge CD is equal to the length AE of the base ABFE, the wedge is evidently equal to half a parallelopipedon ABGCHDEF, having the same base and altitude as the wedge; and this will always be true, whether the end ABC of the wedge be perpendicular to its base ABFE, or inclined as ABI, since parallelopipedons, standing on the same base, and between the same parallels, are equal to each other, (EUCLID 29 *of* XI.) But when the edge CD of the wedge is longer or shorter than the base, by any quantity DK or CI, it is evident that the wedge will be greater

breadth of the base, and that product by the perpendicular height of the wedge; and one-sixth of the last product will be the solidity.

Note. To find the surface of a wedge.—The base is a right-angled parallelogram, the two ends are triangles, and the two sides are parallelograms; if the areas of each of these be found separately, their sum will be the whole surface of the wedge.

Example 1. The perpendicular height O I of a wedge is 24·8 inches, the length C K of the edge 110 inches, the length of the base A E 70 inches, and its breadth A B 30 inches; what is the solidity?

(A E × 2) + C K = (70 × 2) + 110 = 250, sum.

Then (250 × 30 × 24·8) ÷ 6 = 31000 cubic inches = 17·9398 feet, the solidity.

---

greater or less than the half parallelopipedon aforesaid, by a pyramid whose base is E F D, or A B C, and perpendicular altitude D K or C I. Let the length A E of the base of the wedge be repreſented by L, and the breadth A B by B; call the length of the edge l, and the perpendicular height O I, b. Then, the solidity of the wedge will be expressed by $\frac{BL}{2} \times b$ or $\frac{Bb}{2} \times L = \frac{BLb}{2}$, when the edge is of the same length as the base L. the solidity of the pyramid A B C I will be expressed by $\frac{Bb}{2} \times \frac{L-l}{3}$ when the length I D of the edge is less than that of the base; and the solidity of the pyramid E F D K will be expressed by $\frac{Bb}{2} \times \frac{l-L}{3}$, when the length of the edge C K is greater than that of the base. Hence, $\frac{BLb}{2} + \left(\frac{Bb}{2} \times \frac{l-L}{3}\right) = \frac{2L+l}{6} \times Bb$, the rule when the edge is longer than the base, and $\frac{BLb}{2} - \left(\frac{Bb}{2} \times \frac{L-l}{3}\right) = \frac{2L+l}{6} \times Bb$, the rule when the edge is shorter than the base.

2. Required the solidity of a wedge, whose altitude is 14 inches, its edge 21 inches, and the length and breadth of its base 32 and 4·5 inches?

Ans. $892\frac{1}{2}$ inches.

3. Required the solidity of a wedge, whose altitude is 2 feet 4 inches, length of the edge 3 feet 6 inches, length of the base 5 feet 4 inches, and breadth 9 inches? *Ans.* 4·131944, &c. feet.

PROBLEM X.

To find the Solidity of a Prismoid.

RULE*. 1. To the greater length, add half the less length, and multiply the sum by the breadth of the greater end.

2. To

---

* *Demonstration.* The prismoid is evidently composed of two wedges, whose bases are equal to the bases of the prismoid, and their height equal to the height of the prismoid. Let L equal the length of the greater base AB, which of course will be the length of the edge of the less wedge; l equal the length of the less base EF, or the edge of the greater wedge: B the breadth of the greater base AC, b the breadth of the less base EG, and b the common height. Then the solidity of the wedge, whose base is ABDC, will be expressed by $\frac{2L+l}{6} \times Bb$, and the solidity of the wedge whose base is EFHG by $\frac{2l+L}{6} \times bb$; hence the solidity of the whole prismoid will be $\frac{2L+l}{6} \times Bb + \frac{2l+L}{6} \times bb = \overline{\overline{L+\frac{l}{2}} \times B; + \overline{l+\frac{L}{2}} \times b;} \times \frac{b}{3}$, which is the rule, exactly.

This Rule is demonstrated Section II. Prob. XIV. of *Emerson's Fluxions*, and in a similar manner at p. 179, second edition, of *Simpson's Fluxions*, also in *Holliday's Fluxions*, p. 302.

Note. If $\frac{L+l}{2} = M$, and $\frac{B+b}{2} = m$, the above Rule becomes $\overline{BL + bl + 4Mm;} \times \frac{b}{6}$, that is, *to the sum of the areas*

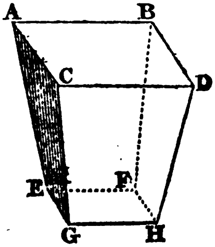
of

2. To the less length, add half the greater length, and multiply the sum by the breadth of the less end.

3. Add the two preceding products together, and multiply the sum by one-third part of the height, and the result will be the solidity.

Note. The surface of a prismoid may be obtained by adding the areas of its four sides and two ends into one sum; and these consist of trapezoids and parallelograms.

Example 1. Let ABCDEFGH be a prismoid, the length AB of the greater end being 38 inches, and its breadth AC 16 inches; the length GH of the less end 30 inches, and its breadth GE 12 inches; what is the solidity, if the perpendicular height be 6 feet?



$$(AB + \tfrac{1}{2}GH) \times AC = (38 + 15) \times 16 = 848.$$
$$(GH + \tfrac{1}{2}AB) \times GE = (30 + 19) \times 12 = 588.$$

Sum = 1436.

Then $1436 \times (72 \div 3) = 34464$ cubic inches $= 19\cdot94\frac{4}{9}$ feet, the solidity.

2. One end of a prismoid is a square, each side of which is 13; the other, a right-angled parallelogram, whose length is 12, and breadth 5; what is the solidity, the perpendicular height being 20?

Ans. $2263\frac{1}{3}$.

---

of the two ends, add four times the area of a section parallel to, and equally distant from, both ends; this last sum, multiplied into one-sixth of the height, will give the solidity.

It is shewn, in Proposition III, p. 456, Part IV, of Dr. *Hutton's Mensuration*, quarto edition, that this last Rule is true, for all frustums whatever, and for all solids whose parallel sections are similar figures. And Mr. *Moss*, in his *Gauging*, p. 175, third edition, says, that it is *nearly* true, let the form of the solid be what it will.

3. In the neighbourhood of *Newcastle*, and in the county of *Durham*, the coals are carried from the mines on an iron rail-way, in a kind of waggon, in the form of a prismoid. The length at the top is generally about 6 feet 9½ inches, and breadth 4 feet 7 inches; at the bottom the length is 3 feet 5 inches, and breadth 2 feet 5½ inches; and the perpendicular depth 3 feet 11¼ inches. Required the solidity,

Ans. 126340·59375 cubic inches = 73·11376 feet.

4. How many gallons of water, reckoning 282 cubic inches to a gallon, are contained in a canal 304 feet by 20 at top, 300 feet by 16 at bottom, and 6 feet deep? *Ans.* 166590·6383 gallons.

PROBLEM XI.

To find the Solidity of a CYLINDROID; *or the Frustum of an elliptical Cone.*

RULE I.

1. To the longer diameter of the greater end, add half the longer diameter of the less end, and multiply the sum by the shorter diameter of the greater end.

2. To the longer diameter of the less end, add half the longer diameter of the greater end, and multiply the sum by the shorter diameter of the less end.

3. Add the two preceding products together, and multiply the sum by ·2618 (one-third of ·7854), and then by the height, the last product will be the solidity.

RULE II*. To the sum of the areas of the two ends, add four times the area of a section parallel to, te, and equally distant from, both ends; the last sum, multiplied by one-sixth of the height, will give the solidity.

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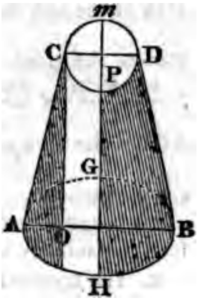
* The first Rule, which is the same as that given for the prismoid, was introduced into this work by Mr. *Hawney*, from *Everard's Gauging.* It is, in reality, the same as the second Rule, when the middle section between the two ends, is similar to the two

Example 1. Let ABCD be a cylindroid, the bottom end of which is an ellipsis, whose two diameters are 44 and 14 inches, the top end a circle, whose diameter is 26 inches; if the cylindroid be 9 feet high, what is its solidity?



By Rule I.

$$(AB + \tfrac{1}{2}CD) \times OH = (44 + 13) \times 14 = 798$$
$$(CD + \tfrac{1}{2}AB) \times mP = (26 + 22) \times 126 = 1248$$
$$\text{Sum} = 2046$$

Then $(2046 \div 144) \times \cdot2618 \times 9 = 33\cdot477675$ feet, the solidity.

---

two ends; but Mr. *Moss* has shewn, in Prob. VI. Sect. VIII. of his *Gauging*, that the middle section can never be an ellipsis, or similar to the two ends, unless the parallel ends be similar ellipses, viz. the transverse and conjugate diameters at each end respectively parallel to each other, and this circumstance can never happen, but when the solid is the frustum of an elliptical cone. The Rule therefore is only an approximation, and not strictly true, when applied to a figure in which one end is circular and the other elliptical, as in Example 1.

By Rule II.

$26^2 \times {\cdot}7854 = 530{\cdot}9304$ area of the less end.

$44 \times 14 \times {\cdot}7854 = 483{\cdot}8064$ area of the greater end.

$\frac{44+26}{2} \times \frac{14+26}{2} \times {\cdot}7854 \times 4 = 2199{\cdot}12$ 4 times the area of the middle section*.

Sum $= 3213{\cdot}8568$ square inches.

Then $(3213{\cdot}8568 \div 144) \times (9 \div 6) = 33{\cdot}477675$ feet, the solidity as before.

2. The transverse diameter of the greater base of a cylindroid is 13, conjugate 8; the transverse diameter of the less base 10, and its conjugate 5·2. If the length of the cylindroid be 120, what is the solidity. *Ans.* 1203·2328.

3. I desire to know what quantity of water an elliptical bath will hold, the longer diameter at the top being 12 feet, and shorter 7; the longer diameter at the bottom 10 feet, and the shorter 6 feet; and the depth 4 feet; reckoning 282 cubic inches to a gallon? *Ans.* 1379·6303 gallons.

PROBLEM XII.

To find the Solidity of a Sphere or Globe.

RULE I. Multiply the cube of the diameter by ·5236, and the product will be the solidity†.

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* Half the sum of A B and C D is 35, the greater diameter of the middle section; and half the sum of G H and E F is 20, the less diameter. Supposing this section to be an ellipsis, four times its area will be as above; but if the section be any other curve, this area will not be perfectly correct, and, of course, the solidity will be incorrect likewise.

† The solidity of a sphere is two-thirds of the solidity of its circumscribing cylinder. *Keith's Geom.* 21 of X.

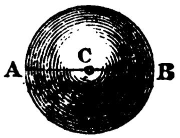
RULE II. Multiply the diameter of the sphere by its circumference, and the product will be the convex superficies *; and if this superficies be multiplied by one-sixth part of the diameter, the result will be the solidity.

Example 1. What is the solidity of a globe, whose diameter AB is 20 inches?

By Rule I.

$AB^3 \times \cdot5236 = 20^3 \times \cdot5236 = 4188\cdot8$ cubic inches = 2·424 feet, the solidity.

By Rule II.



$AB \times 3\cdot1416 = 20 \times 3\cdot1416 = 62\cdot832$ inches, the circumference of the sphere; and $62\cdot832 \times AB = 62\cdot832 \times 20 = 1256\cdot64$ square inches, the convex superficies.

Hence $1256\cdot64 \times \frac{1}{6}AB = 1256\cdot64 \times \frac{20}{6} =$ 4188·8 cubic inches, the solidity as before.

2. The diameter of the earth is 7970 miles, what is its surface in square miles, and solidity in cubic miles?

Ans. { Surface 199557259·44 miles.
Solidity 265078559622·8 miles.

3. The circumference of a sphere is 1, what are its solidity and superficies?

Ans. { Solidity ·0168868.
Superficies ·3183099.

---

* The curve surface of a sphere, or of any zone or segment thereof, is equal to the curve surface of a cylinder, of the same height, when the diameter of the base of the cylinder is equal to the diameter of the sphere. Therefore, if the circumference of the sphere be multiplied by the height of the zone or segment, the product will be the curve surface of such zone or segment.

4. What are the solid and superficial contents of a globe, whose diameter is 30?

Ans. { Solidity 14137·2.
Superficies 2827·44.

5. The circumference of a globe is 50·3, what are its solidity and superficies?

Ans. { Solidity 2149·073728.
Superficies 805·3527.

6. A globe, a cube, a cylinder,
All three in surface equal are*,
In solidity what do they differ?

* Viz. 3·1416 each.

Ans. The solidity of the globe is ·5236, the cube ·378877, and cylinder ·4275176; so that of all solids, under the same superficies, the globe is the greatest.

PROBLEM XIII.

To find the Solidity of the Segment of a Sphere.

RULE 1 *. From three times the diameter of the sphere, subtract twice the height of the segment;

---

* *Demonstration.* Let A C *m* represent a triangular pyramid, whose side *m* A is infinitely small. The sphere may be considered to be constituted of an infinite number of such pyramids A C *n*, *n* C *r*, &c. whose bases compose the spherical surface, and their altitudes are equal to the radius of the sphere, the common vertex being the centre C.

Hence it follows, that the solidity of the sphere, or any sector thereof, is equal to a pyramid, the area of whose base is equal to the spherical surface, and its height equal to the radius of the sphere.

Let O D, the height of the segment A D B, be represented by *b*, and the diameter D E by D. Then the circumference of the sphere will be 3·1416 D, and the surface of the segment (see the Note to Prob. XII. page 119), 3·1416 D *b*. Also the solidity of the spherical sector A C B D will be $3{\cdot}1416\,\text{D}b \times \frac{1}{3}\,\text{CD} = 3{\cdot}1416\,\text{D}b \times \frac{1}{6}\,\text{D} = {\cdot}5236\,\text{D}^2 b$

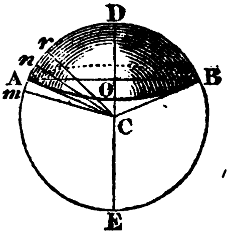
By Note 8, p. 47, $\text{EO} \times \text{OD} = \text{AO}^2$, viz. $(\text{D} - b) \times b = \text{AO}^2$; but $\text{AO}^2 \times 3{\cdot}1416 \times \frac{1}{3}\,\text{OC} = (\text{D} - b) \times b \times 3{\cdot}1416 \times \frac{1}{3}\left(\frac{\text{D}}{2} - b\right) = (\text{D} - b) \times b \times {\cdot}5236 \times (\text{D} - 2b) =$ $\cdot(\text{D}^2 b -$

multiply the remainder by the square of the height, and that product by ·5236, the result will be the solidity.

Rule II. To three times the square of the radius of the segment's base, add the square of its height; multiply this sum by the height, and the product by ·5236, the result will be the solidity.

Example 1. Let A B D O be the segment of a sphere, of which the diameter A B of the base is 16 inches, and the height O D 4 inches, what is the solidity?

By Rule I.



By Note 8, p. 47, $\mathrm{EO} = \mathrm{AO}^2 \div \mathrm{OD} = 8^2 \div 4 = 16$, and $\mathrm{EO} + \mathrm{OD} = \mathrm{DE} = 16 + 4 = 20$.

Then $(3\,\mathrm{D} - 2\,\mathrm{OD}) \times \mathrm{OD}^2 \times \cdot 5236 = (60 - 8) \times 4^2 \times \cdot 5236 = 832 \times \cdot 5236 = 435{\cdot}6352$ cubic inches, the solidity.

$(\mathrm{D}^2 b - 3\,\mathrm{D} b^2 + 2 b^3) \times \cdot 5236$, the solidity of the cone A C B. Now the solidity of the cone, taken from the solidity of the sector, leaves the solidity of the segment, viz. $(\cdot 5236\,\mathrm{D}^2 b) - (\mathrm{D}^2 b - 3\,\mathrm{D} b^2 + 2 b^3) \times \cdot 5236 = (3\,\mathrm{D} b^2 - 2 b^3) \times \cdot 5236 = (3\,\mathrm{D} - 2 b) \times b^2 \times \cdot 5236$, which is the first Rule; and it is evident that this Rule will be true, whether the segment be greater or less than the hemisphere, or whatever be the magnitude of b, provided it be less than D.

If $r = \mathrm{AO}$, the radius of the segment's base, then $(\mathrm{D} - b) \times b = r^2$, therefore $\mathrm{D} = \dfrac{r^2 + b^2}{b}$, for D, in the above Rule, substitute its value, and the Rule becomes $\left(\dfrac{3 r^2 + 3 b^2}{b} - 2 b\right) \times b^2 \times \cdot 5236 = (3 r^2 + b^2) \times b \times \cdot 5236$, which is the second Rule, and is very useful when the diameter of the sphere is not given.

By Rule II.

$(3\,\mathrm{AO}^2 + \mathrm{OD}^2) \times \mathrm{OD} \times \cdot 5236 = (192 + 16) \times 4 \times \cdot 5236 = 832 \times \cdot 5236 = 435\cdot 6352$ cubic inches, the solidity as before.

2. What is the solidity of the segment of a sphere, whose height is 9, and the diameter of its base 20? *Ans.* 1795·4244.

3. What is the solidity of the segment of a sphere, whose diameter is 20 feet, and the height of the segment 5 feet? *Ans.* 654·5 feet.

4. The diameter of a sphere is 21, what is the solidity of a segment thereof, whose height is 4·5? *Ans.* 572·5566.

5. The diameter of the base of a segment of a sphere is 28, and the height of the segment 6·5; required the solidity? *Ans.* 2144·99285.

PROBLEM XIV.

To find the Solidity of the Frustum or Zone of a Sphere.

RULE I[*]. To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance,

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[*] *Demonstration.* Call the height of the greater segment H, and the height of the less b; also R the radius of the greater base, and r that of the less: then it is evident the difference between the solidities of these two segments will be the solidity of the zone. Hence $\overline{3\,\mathrm{R}^2 + \mathrm{H}^2} \times \mathrm{H} \times \cdot 5236; - \overline{3\,r^2 + b^2} \times b \times \cdot 5236 = (3\,\mathrm{R}^2\mathrm{H} + \mathrm{H}^3; - 3\,r^2 b + b^3) \times \cdot 5236$. Put $a = \mathrm{H} - b$ the breadth of the zone, and D the diameter of the sphere: then, by the property of the circle $\overline{\mathrm{D} - \mathrm{H}} \times \mathrm{H} = \mathrm{R}^2$, and $\overline{\mathrm{D} - b} \times b = r^2$ from the former of these $\mathrm{D} = \frac{\mathrm{R}^2 + \mathrm{H}^2}{\mathrm{H}}$, and from the latter $\mathrm{D} = \frac{r^2 + b^2}{b}$; therefore $\frac{\mathrm{R}^2 + \mathrm{H}^2}{\mathrm{H}} = \frac{r^2 + b^2}{b}$, and from above $a = \mathrm{H} - b$. Exterminate the values of

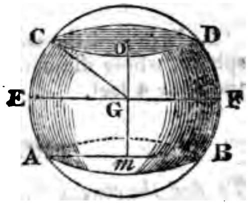
tance, or the height of the zone; this sum multiplied by the height of the zone, and the product again by 1·5708, will give the solidity.

Rule II. *For the middle zone of a sphere.*

To the square of the diameter of the end, add two-thirds of the square of the height; multiply this sum by the height, and then by ·7854, for the solidity.

Or, from the square of the diameter of the sphere, subtract one-third of the square of the height of the middle zone; multiply the remainder by the height, and then by ·7854, for the solidity.

Example 1. Required the solidity of the zone of a sphere E F D C, whose greater diameter E F is 20 inches, less diameter C D 15 inches, and distance between the ends, or height G O, 10 inches.



$$(EG^2 + CO^2) + \tfrac{1}{3}\,GO^2 = (100 + 56{\cdot}25) + \frac{100}{3} = 189{\cdot}58\tfrac{1}{3}.$$

---

of H and b, and the above theorem $(3\,R^2\,H + H^3;\ - \overline{3\,r^2 b + b^3}) : \times \cdot 5236$, will become $\left(R^2 + r^2 + \dfrac{a^2}{3}\right) \times a \times 1{\cdot}5708$.

If one of the radii pass through the centre, as in the zone E F D C, then $R^2 = \dfrac{D^2}{4} = CO^2 + GO^2 = r^2 + a^2$; hence the last theorem becomes $(r^2 + \tfrac{2}{3}\,a^2) \times a \times 3{\cdot}1416 = (\tfrac{1}{4}\,D^2 - \tfrac{1}{3}\,a^2) \times a \times 3{\cdot}1416$.

Hence $(r^2 + \tfrac{2}{3}\,a^2) \times a \times 6{\cdot}2832 = (\tfrac{1}{4}\,D^2 - \tfrac{1}{3}\,a^2) \times a \times 6{\cdot}2832$ *will express the solidity of the middle zone* C D B A, being double of the former, where a is half the altitude, and $r =$ half the diameter of each end. Put A for the whole altitude, and $d = 2\,r$ the diameter of each end, and the theorems become $(d^2 + \tfrac{2}{3}\,A^2) \times A \times \cdot 7854 = (D^2 - \tfrac{1}{3}\,A^2) \times A \times \cdot 7854$.

Then 189·58$\frac{1}{3}$ × 10 × 1·5708 = 5687·5 × ·5236 = 2977·975 cubic inches, the solidity required.

2. Required the solidity of the middle zone A B D C of a sphere, whose diameter E F is 22 inches; the top and bottom diameters C D and A B of the zone being each 16·971 inches, and the height M O 14 inches? *Ans.* 4603·4912 inches.

3. Required the solidity of a zone, whose greater diameter is 12, less diameter 10, and height 2.
Ans. 195·8264.

4. Required the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 feet, and the height of the zone 4 feet.
Ans. 61·7848 feet.

5. Required the solidity of the middle zone of a sphere, whose diameter is 5 feet, and the height of the zone 4 feet. *Ans.* 61·7848 feet.

PROBLEM XV.

To find the convex Surface of any Segment, or Zone, of a Sphere.

Rule. Multiply the circumference of the whole sphere by the height of the segment, or zone; and the product will be the convex surface.

Example 1. If the diameter of the earth be 7970 miles, the height of the frigid zone will be 252·361283 miles; required its surface?
Ans. 6318761·107182216 miles.

2. If the diameter of the earth be 7970 miles, the height of the temperate zone will be 2143·6235535 miles; required its surface?
Ans. 53673229·812734532 miles.

3. If the diameter of the earth be 7970 miles, the height of the torrid zone will be 3178·030327 miles; required its surface? *Ans.* 79573277·600166504.

Note.

Note. The surfaces of the two frigid zones, by Example 1, are - -	6318761·107182216 6318761·107182216
The surfaces of the two temperate zones, by Example 2, are - - - -	53673229·812734532 53673229·812734532
The torrid zone, Example 3, is - -	79573277·600166504
Surface of the whole globe, agreeing exactly with Example 2, Prob. XII. page 119 - - - - - - -	199557259·440000000

PROBLEM XVI.

To find the Solidity of a circular Spindle.

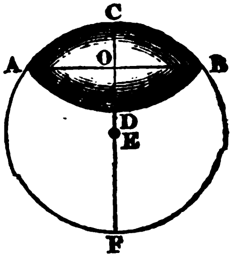
Rule. Find the distance of the chord of the generating* circular segment from the centre of the circle, (*Note* 8, *Prob.* XII, *page* 47) and also the area of this segment (*by Prob.* XVII. *page* 59).

Then, from $\frac{1}{3}$ of the cube of half the length of the spindle, or half chord of the segment, subtract the product of the central distance, and half the area of the segment; the remainder, multiplied by 12·5664, will give the solidity.

Note. To find the superficies. Find the distance of the chord of the generating circular segment from the centre of the circle, and the length of the arc of this segment, (*Prob.* XII. p. 46.)

Then, from the product of the length of the spindle, or chord of the segment, and the radius of the circle, subtract the product of the central distance, and the length of the arc; the remainder, multiplied by 6·2832, will give the surface required.

Example 1. The length A B of a circular spindle is 40 inches, and its greatest, or middle, diameter C D 30 inches; required the solidity and superficies.



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* See Definition 18, p. 95.

First. $AO^2 \div OC = FO = 20^2 \div 15 = 26\frac{2}{3}$ (*Note* 8, *Prob.* XII. *page* 47), $FO + OC = CF = 26\frac{2}{3} + 15 = 41\frac{2}{3}$, the diameter of the circle; $\frac{1}{2} CF - OC = EC - OC = 5\frac{5}{6} = EO$, the *central distance.*

Second. $CO \div CF = 15 \div 41\frac{2}{3} = \cdot 36$ the *area seg.* corresponding to which is ·254550, which, multiplied by the square of $41\frac{2}{3}$, produces 441·92708, the *area of the generating segment* ACB, the half of which is 220·96354.

Third. $(20^3 \div 3) - (5\frac{5}{6} \times 220\cdot96354) = 1377\cdot71268$, which, multiplied by 12·5664, produces 17312·88862 cubic inches, the solidity.

To find the Superficies.

First. The centre distance $EO = 5\frac{5}{6}$, as above. $\sqrt{AO^2 + OC^2} = \sqrt{20^2 + 15^2} = \sqrt{625} = 25 = AC$, the chord of half the arc ACB.

$(25 \times 8) - 40 = 160$ and $160 \div 3 = 53\frac{1}{3}$, length of the arc ACB.

Second. $(40 \times 20\frac{5}{6}) - (5\frac{5}{6} \times 53\frac{1}{3}) = 522\frac{2}{9}$, which, multiplied by 6·2832, produces 3281·226, the superficies required.

2. What are the solidity and superficies of a circular spindle, the length AB being 48, and the middle diameter CD 36?

Ans. $\begin{cases} \text{Solidity} = 29916\cdot6714. \\ \text{Superficies} = 4724\cdot9664. \end{cases}$

PROBLEM XVII.

To find the Solidity of the middle Frustum, or Zone, of a circular Spindle.

RULE 1. Find the distance of the centre of the middle frustum, from the centre of the circle, as in Problem XVI.

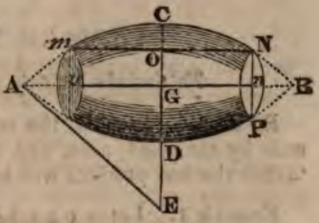
2. Find the area of a segment of a circle, the chord of which is equal to the length of the frustum, and height half the difference between its

greatest and least diameters; to which add the rectangle of the length of the frustum and half its least diameter; the result will be the generating surface.

3. From the square of the radius, subtract the square of the central distance, the square-root of the remainder will give half the length of the spindle.

4. From the square of half the length of the spindle, take $\frac{1}{3}$ of the square of half the length of the middle frustum, and multiply the remainder by the said half length.

Multiply the central distance by the generating surface, and subtract this product from the preceding; the remainder, multiplied by 6·2832, will give the solidity.



Example 1. What is the solidity of the middle frustum of a circular spindle, the length m N being 40 inches, the greatest diameter C D 32 inches, and the least diameter N P 24 inches?

First. $m\,\text{O}^2 \div \text{O C} = \text{E O} + \text{E C} = 20^2 \div 4 = 100$, (*Note* 8, *Prob.* XII, *page* 47), and $\text{E O} + \text{E C} + \text{O C} = 100 + 4 = 104$, the diameter of the circle; also $\text{E C} - \text{G C} = 52 - 16 = 36 = \text{E G}$, the *central distance.*

Second. $\frac{1}{2}\,(\text{D C} - \text{P N}) = \frac{1}{2}\,(32 - 24) = 4 = \text{C O}$.

Then $4 \div 104 = {\cdot}038\frac{6}{13}$, the *area seg.* corresponding to which is ·009940; which, multiplied by the square of 104, produces 107·51104, the area of the segment m C N; and $m\,\text{N} \times \frac{1}{2}\,\text{N P} = 40 \times 12 = 480$, area of the rectangle m N n v. Then $107{\cdot}51104 + 480 = 587{\cdot}51104$, the *generating surface* v m C N n.

Third. $\sqrt{AE^2 - EG^2} = \sqrt{52^2 - 36^2} = \sqrt{1408} = 8\sqrt{22} = AG$, half the length of the spindle.

Fourth. $(AG^2 - \frac{1}{3}mo^2) \times mo = \left(1408 - \frac{400}{3}\right) \times 20 = 25493\frac{1}{3}$.

Then $36 \times 587{\cdot}51104 = 21150{\cdot}39744$, and $(25493\frac{1}{3} - 21150{\cdot}39744) \times 6{\cdot}2832 = 27287{\cdot}5347$ cubic inches, the solidity.

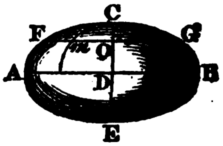
2. What is the content, in ale gallons, of a cask in the form of the middle frustum of a circular spindle, the bung diameter CD being 60 inches, the head diameter NP 36 inches, and the length *m*N or *v n* 80 inches? *Ans.* 174890·9116 cubic inches, or 620·18052 gallons.

PROBLEM XVIII.

To find the Solidity of a Spheroid.

Rule *. Multiply the square of the revolving axis by the fixed axis, and this product again by ·5236, the last product will be the solidity.

Example 1. Let ACBE be a *prolate* spheroid, whose longer axis AB is 55 inches, and shorter axis CE 33: required the solidity.



$CE^2 \times AB \times {\cdot}5236 = 33^2 \times 55 \times {\cdot}5236 = 31361{\cdot}022$ cubic inches, the solidity.

2. What is the solidity of an *oblate* spheroid, whose longer axis is 55 inches, and shorter axis 33 inches? *Ans.* 52268·37 inches.

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* This Rule is demonstrated at p. 299, of *Holliday's Fluxions*; p. 173, vol. i. of *Simpson's Fluxions*; p. 46, of *Dealtry's Fluxions*, &c.

3. The axes of an oblong or *prolate* spheroid are 50 and 30; required the solidity. *Ans.* 23562.

4. The axes of an *oblate* spheroid are 50 and 30; required the solidity. *Ans.* 39270.

PROBLEM XIX.

To find the Solidity of the Segment of a Spheroid, the Base of the Segment being parallel to the revolving Axis of the Spheroid.

RULE [*]. From three times the fixed axis, subtract twice the height of the segment, multiply the remainder by the square of the height, and that product by ·5236.

Then, the square of the fixed axis, is to the square of the revolving axis; as the product, found above, is to the solidity of the spheroidal segment.

Example 1. In an *oblate* spheroid, the transverse or revolving axis A B is 100, the conjugate or fixed axis E C is 60, and the height O C of the segment F C G is 10; required the solidity.

$(3\,EC - 2\,CO) \times CO^2 \times {\cdot}5236 = (180 - 20) \times 10^2 \times {\cdot}5236 = 8377{\cdot}6$, solidity of a spherical segment, whose radius is *m* O, and height C O.

Then $60^2 : 100^2 :: 8377{\cdot}6 : 23271\frac{1}{9}$, the solidity of the elliptical segment F C G.

2. In a *prolate* spheroid, the transverse or fixed axis is 100, the conjugate or revolving axis is 60, and the height of the segment 10; required the solidity. *Ans.* 5277·888.

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[*] This Rule is nearly the same as that given in Problem XIII, page 120. For the solidity of the whole sphere, is to the solidity of the whole spheroid; as any segment of the sphere, is to a like segment of the spheroid. Now $EC^2 : AB^2 :: mO^2$ the radius of the base of a circular segment, whose height is $OC : FO^2$ the radius of the base of an elliptical segment, whose height is O C. *Emerson's Conic Sections*, Book I. Prop. XIX.

3. The axes of a prolate spheroid are 50 and 30; what is the solidity of a segment whose height is 5, and its base parallel to the conjugate or revolving axis? *Ans.* 659·736.

4. The axes of an oblate spheroid are 50 and 30; what is the solidity of a segment whose height is 6, and its base parallel to the transverse or revolving axis? *Ans.* 4084·08.

5. If the axes of a prolate spheroid be 10 and 6; required the area of a segment whose height is 1, and its base parallel to the conjugate or revolving axis? *Ans.* 5 277888.

6. The axes of a prolate spheroid are $36\frac{2}{3}$ and 18; what is the solidity of a segment whose height is $6\frac{1}{2}$, and its base parallel to the conjugate or revolving axis? *Ans.* 517·1306.

PROBLEM XX.

To find the Solidity of the middle Zone of a Spheroid, the Diameter of the Ends being perpendicular to the fixed Axis, the middle Diameter, and that of either End being given, together with the Length of the Zone.

Rule *. To twice the square of the middle diameter, add the square of the diameter of the end;

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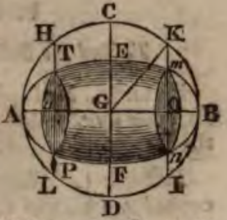
* Let f = AB the fixed axis } for the oblong spheroid.
r = EF the revolving axis }
b = vo the height of the middle zone or frustum.
D = KI the diameter of one end of the spherical zone.
d = mn the corresponding diameter of the spheroidal zone.

By Prob. XIV. Rule 2, the solidity of the middle zone of the sphere is $D^2 + \frac{2b^2}{3} \times b \times \cdot 7854 = 3D^2 + 2b^2; \times b \times$ 2618, but $Ko^2 = GK^2 - Go^2$, viz. $\frac{D^2}{4} = \frac{f^2}{4} - \frac{b^2}{4}$, therefore $D^2 = f^2 - b^2$, hence the solidity of the spherical zone = $3f^2$

multiply the sum by the length of the zone, and the product again by ·2618 for the solidity.

Note. This rule is useful in gauging. A cask in the form of the middle zone of a prolate spheroid, is, by gaugers, called a cask of the *first* variety.



Example 1. What is the solidity of the middle zone T E m n F P of a prolate spheroid, the diameter $m\,n$ or T P of the end being 36 inches, the middle diameter E F 60 inches, and the length $v\,o$ 80 inches?

$(2\,\text{EF}^2 + mn^2) \times vo = (7200 + 1296) \times 80 = 679680$; which, multiplied by ·2618, produces 177910·224 cubic inches, the solidity required.

2. Required the solidity of the middle zone of an oblate spheroid, the middle diameter being 100, the diameter of the end 80, and the length 36?

Ans. 248814·72.

3. Required the content, in ale gallons, of a spheroidal cask, whose length is 40 inches; the bung diameter being 32, and head diameter 24 inches. A gallon of ale being 282 cubic inches? *Ans.* 97·44.

$\overline{3f^2 - b^2}; \times b \times {}^{\cdot}2618$. But $\cdot 5236\,f^3 : \cdot 5236\,fr^2 :: \overline{3f^2 - b^2}; \times b \times {}^{\cdot}2618$: the solidity of the spheroidal frustum, viz.

$f^2 : r^2 :: \overline{3f^2 - b^2}; \times b \times {}^{\cdot}2618$: the solidity of the spheroidal frustum. By the property of the ellipsis $\text{CD}^2 : \text{EF}^2 :: \text{KI}^2 : mn^2$, that is,

$$f^2 : r^2 :: f^2 - b^2 : d^2, \text{ hence } f^2 = \frac{r^2 b^2}{r^2 - d^2}.$$

Conseq. $\frac{r^2 b^2}{r^2 - d^2} : r^2 :: \overline{\frac{3 r^2 b^2}{r^2 - d^2} - b^2}; \times b \times {}^{\cdot}2618$: the solidity of the spheroidal frustum $= \overline{2r^2 + d^2}; \times b \times {}^{\cdot}2618$. The rule for the oblate spheroidal zone will be exactly the same, putting r for A B, the revolving axis.

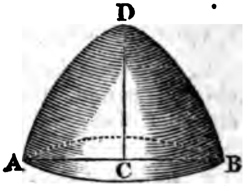
4. Required the content, in wine gallons of a spheroidal cask, whose length is 20 inches; the bung diameter being 16, and head diameter 12 inches. A gallon of wine being 231 cubic inches? *Ans.* 14·869.

PROBLEM XXI.

To find the Solidity of a parabolic Conoid.

Rule*. Multiply the square of the diameter of its base by ·3927, and that product by the height; the last product will be the solidity.

Example 1. What is the solidity of the parabolic conoid ABD, the diameter AB of its base being 36 inches, and its height CD 33 inches?



$AB^2 \times \cdot 3927 \times CD = 36^2 \times \cdot 3927 \times 33 =$ 16794·9936 cubic inches; which, divided by 1728, gives 9·719 cubic feet, the solidity required.

2. What is the solidity of a parabolic conoid whose height is 84, and the diameter of its base 48? *Ans.* 76001·5872.

3. Required the solidity of a paraboloid whose height is 30, and the diameter of its base 40? *Ans.* 18849·6.

4. Required the solidity of a paraboloid whose height is 22·85, and the diameter of its base 32? *Ans.* 9188·55168.

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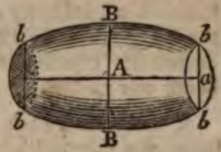
* The solidity of a parabolic conoid, or paraboloid, is equal to *half* the solidity of a cylinder of the same base and height. This is demonstrated in *Holliday's Fluxions*, p. 298; *Simpson's Fluxions*, p. 173; *Dealtry's Fluxions*, p. 45; *Emerson's Fluxions*, p. 283, &c.

PROBLEM XXII.

To find the Solidity of the Frustum of a parabolic Conoid.

RULE *. Multiply the sum of the squares of the diameters of the two ends by the height, and that product by ·3927 for the solidity.

Note. This rule is useful in cask-gauging. For if two equal frustums of a parabolic conoid B *b b* B, be joined together at their greater bases, they form a cask of the *third* variety. The less diameter of the frustum *b b* will be the head diameter of the cask, and the greater diameter B A B will represent the bung diameter.

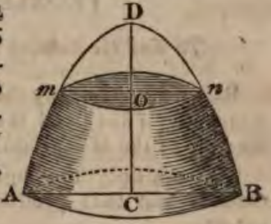


Example 1. The greater diameter A C B of the frustum A *m n* B is 32, the less diameter *m* O *n* = 26, and the height C O = 8; required the solidity.

$\text{A B}^2 = 32^2 = 1024$
$m\,n^2 = 26^2 = 676$

Sum of the squares = 1700

Then 1700 × 8 × ·3927 = 5340·72 cubic inches, the solidity required.



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* Let b = C O the height of the frustum, p = D O the height of the part m D n cut off, then $b + p$ = C D the height of the whole conoid. Put D = A C B, and d = m O n, then the solidity of A D B will be $(\text{D}^2 b + \text{D}^2 p) \times {}^{\cdot}3927$, and the solidity of m D n will be $d^2 p \times {}^{\cdot}3927$, hence the solidity of the frustum A m n B must be $(\text{D}^2 b + \text{D}^2 p - d^2 p) \times {}^{\cdot}3927$. But, by the property of the parabola (*Note* 5, *page* 78) C D : O D :: $\text{A C}^2 : m\,\text{O}^2$, viz. $b + p : p :: \frac{1}{4}\text{D}^2 : \frac{1}{4}d^2$, hence $\text{D}^2 p - d^2 p = b d^2$, substitute this for its value in the expression for the solidity of the frustum, and it becomes $(\text{D}^2 + d^2) \times b \times {}^{\cdot}3927$, which is the rule.

2. Required the solidity of the frustum of a parabolic conoid, whose greater diameter is 30, less diameter 24, and the height 9? *Ans.* 5216·6268.

3. Required the solidity of the frustum of a paraboloid, the diameter of the greater end being 60, that of the less end 48, and the height 18?
Ans. 41733·0144.

4. There is a cask in the form of two equal frustums of a parabolic conoid; the length is 40 inches, the bung diameter 32, and head diameter 24. Required its content in ale gallons? 282 cubic inches being one gallon. *Ans.* 89·1234 gallons.

5. There is a cask in the form of two equal frustums of a paraboloid; the length is 20 inches, the bung diameter 16, and head diameter 12. Required the content in wine gallons? 231 cubic inches being one gallon. *Ans.* 13·6 gallons.

PROBLEM XXIII.

To find the Solidity of a parabolic Spindle.

RULE *. Multiply the square of the middle diameter by ·7854, and that product by the length; then $\frac{8}{15}$ of the last product will be the solidity.

Or, multiply the square of the diameter by ·41888, and that product by the length, for the solidity.

Example 1. Let A B C D be a parabolic spindle, of which the middle diameter C D is 36 inches, and the length A B 99 inches; required the solidity.

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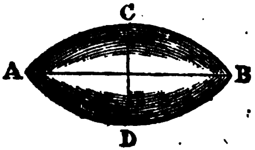
* The solidity of a parabolic spindle is equal to $\frac{8}{15}$ of its circumscribing cylinder. This is demonstrated in *Holliday's Fluxions*, p. 304; *Emerson's Fluxions*, p. 284; *Simpson's Fluxions*, p. 177, &c.



$CD^2 \times {\cdot}7854 \times 99 = 36^2 \times {\cdot}7854 \times 99 = 100769{\cdot}9616$.

Then $(100769{\cdot}9616 \times 8) \div 15 = 53743{\cdot}97952$ cubic inches $= 31{\cdot}10184$ cubic feet, the solidity.

Or, $36^2 \times {\cdot}41888 \times 99 = 53743{\cdot}97952$ cubic inches, the solidity.

2. The length of a parabolic spindle is 60, and the middle diameter 34; what is the solidity?

Ans. 29053·5168.

3. The length of a parabolic spindle is 9 feet, and the middle diameter 3 feet; what is the solidity?

Ans. 33·92923.

4. What is the solidity of a parabolic spindle, whose length is 40, and middle diameter 16?

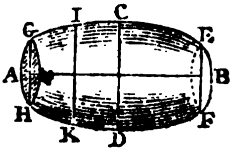
Ans. 4289·3312.

PROBLEM XXIV.

To find the Solidity of the middle Frustum of a parabolic Spindle.

Rule. To twice the square of the middle diameter, add the square of the diameter of the end; and from the sum subtract four-tenths of the square of the difference between these diameters: the remainder multiplied by the length, and that product by ·2618, will give the solidity.

Note. This rule is useful in cask gauging. A cask in the form of the middle frustum of a parabolic spindle, is called by gaugers a cask of the *second* variety; and is the most common of any of the varieties.



Example 1. Required the solidity of the middle frustum of a parabolic spindle E F G H, the length A B being 20, the greatest diameter C D 16, and the least diameter E F or G H 12?

(2 CD²

$(2\,\text{CD}^2 + \text{EF}^2) - (\cdot 4 \times \overline{\text{CD} - \text{EF}}|^2) = (512 + 144) - 6\cdot 4 = 649\cdot 6$, hence $649\cdot 6 \times 20 \times \cdot 2168 = 3401\cdot 3056$, the solidity required.

2. The bung diameter CD of a cask is 32 inches, head diameter EF 24 inches, and length AB 40 inches; required its content in ale gallons: 282 cubic inches being one gallon. *Ans.* 96·4909 gallons.

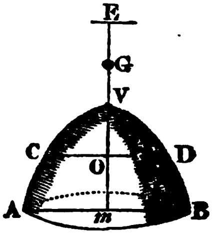
3. The bung diameter CD of a cask is 36 inches, the head diameter EF 20 inches, and length AB 36 inches; what is its content in wine gallons? 231 cubic inches being one gallon. *Ans.* 117·89568 gallons.

PROBLEM XXV.

To find the Solidity of an hyperbolic Conoid.

RULE[*]. To twice the height of the solid, add 3 times the transverse axis, multiply the sum by the square of the radius of the base, and that product by the height, and the last product by ·5236; the result, divided by the sum of the height and transverse axis, will give the solidity.

Example 1. What is the solidity of an hyperbolic conoid, whose height mV = 50, the diameter AB of the base = 103·923048, and the transverse axis EV = 100?



$$\frac{(2\,m\text{V} + 3\,\text{VE}) \times \text{A}m^2 \times m\text{V} \times \cdot 5236}{m\text{V} + \text{VE}} = \text{the}$$

solidity. Now, $(100 + 300) \times \dfrac{\overline{103\cdot 923048}|^2}{2} =$

---

[*] This Rule is deduced from p. 305, of *Holliday's Fluxions*.

400 ×

$400 \times 2700 = 1080000$; and $\frac{1080000 \times 50 \times \cdot 5236}{150}$ $= 188496$, the solidity required.

2. Required the solidity of an hyperboloid, whose altitude m V is 10, the diameter A B of the base 24, and the transverse diameter V E $= 30$.

Ans. 2073·456.

3. The altitude of an hyperboloid is 50, the diameter of its base 104, and the transverse axis E V 122·449; what is the solidity? *Ans.* 191847·042.

4. What is the solidity of a parabolic conoid, the altitude m V being 10, the diameter A B of the base 24, and the middle diameter C D $= 6\sqrt{7}$\*?

Ans. The transverse axis V E $= 30$, and the solidity $= 2072 \cdot 664$.

5. The diameter A B of the base of a parabolic conoid A C V D B is 60, the middle diameter C D 35·49648, and the altitude m V $= 50$, what is the solidity? *Ans.* The transverse diameter V E $= 33\frac{1}{3}$, and the solidity $= 56548 \cdot 8$.

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* In this and the following Example, it will be necessary to find the transverse axis, before the rule given can be applied: a general theorem for this purpose may be obtained in the following manner. By property 7, p. 84, we easily derive the following proportion, viz.

$$mV \times mE : Am^2 :: OV \times OE : CO^2.$$

Let the transverse axis V E be represented by x, then $mE = mV + x$, and $OE = OV + x$: hence $mV \times (mV + x) : Am^2 :: OV \times (OV + x) : CO^2$, from which proportion $x = \frac{(CO^2 \times mV^2) \backsim (Am^2 \times OV^2)}{(Am^2 \times OV) \backsim (CO^2 \times mV)}$ a general theorem for finding the transverse axis, when two abscissas and their ordinates are given. But when m V is bisected by the ordinate C O, that is, when C D is a middle diameter, the theorem may be simplified, and V E, the transverse axis, will be equal to $\frac{4CO^2 \backsim Am^2}{2Am^2 \backsim 4CO^2} \times mV$. Hence, if the *middle* diameter, the diameter of the base, and the altitude of a parabolic conoid be given, the *transverse* axis may be found. Or, if the *transverse* axis, the diameter of the base, and the altitude be given, the *middle* diameter may be found.

PRO-

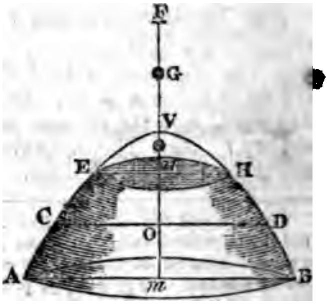
PROBLEM XXVI.

To find the Solidity of the Frustum of an hyperbolic Conoid.

RULE *. To four times the square of the middle diameter, add the sum of the squares of the greatest and least diameters; multiply the result by the altitude, and that product by ·1309, for the solidity.

Example 1. What is the solidity of the frustum ACEHDB of an hyperbolic conoid, whose greatest diameter AB is 96, least diameter EH 54, middle diameter CD 76·4264352, and the altitude *mn* 25?

$4CD^2 + AB^2 + EH^2 = (5841 \times 4) + 9216 + 2916 = 35496$; and $35496 \times 25 \times ·1309 = 116260·66$, the solidity required.



2. What is the solidity of the frustum ACEHDB of an hyperboloid, whose greatest diameter AB is 32 inches, least diameter EH 24 inches; middle diameter CD 2817 inches, and the height *mn* 20 inches?
Ans. 12498·8440808 inches.

3. Required the solidity of the frustum of an hyperbolic conoid, the height being 12, the greatest diameter 10, the least diameter 6, and the middle diameter 8·5.
Ans. 667·59.

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* This Rule is the same as that given in the Note to Prob. X. p. 115, viz. Four times the area in the middle, added to the areas of the two ends, and the sum multiplied by one-sixth of the height; the number ·1309 being the sixth part of ·7854.

PROBLEM XXVII.

To find the Solidity of the middle Frustum of any Spindle, formed by the Revolution of a conic Section about its Axis.

RULE*. To the square of the greatest diameter, add the square of the least, and four times the square of a diameter, taken exactly in the middle between the two; multiply the sum by the length, and that product by ·1309, for the solidity.

Example 1. Required the solidity of the middle frustum EFGH† of any spindle, the length AB being 40, the middle diameter CD 32, the least diameter EF or GH 24, and the diameter IK, in the middle between GH and CD 30·157568?

Ans. 27425·72624.

2. What is the solidity of the middle frustum of a spindle, whose length AB = 20 inches, the middle diameter CD = 16 inches, the diameter at each end 12 inches, and the diameter IK at a quarter of the length 14·5 inches? *Ans.* 3248·938 inches.

PROBLEM XXVIII.

To find the Solidity of a cylindrical Ring‡.

RULE. To the thickness of the ring, add the inner diameter, multiply this sum by the square of the thickness, and the product by 2·4674, for the solidity.

Note. To find the superficies. To the thickness of the ring, add the inner diameter, multiply the sum by the thickness, and that product by 9·8696, for the superficies.

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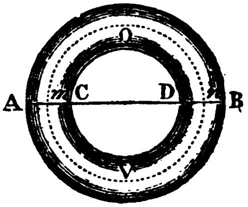
* This Rule is the same as that given in Problem XXVI.

† See the figure to Problem XXIV.

‡ This being only a bent cylinder, the reason of the Rule is obvious.

Example 1. The thickness AC of a cylindrical ring is 3 inches, the diameter CD 8 inches, what are the solidity and superficies.

1. *For the Solidity.*



(AC + CD) × AC^2 = (3 + 8) × 9 = 99, then 99 × 2·4674 = 244·2726 cubic inches, the solidity.

For the Superficies.

(AC + CD) × AC = (3 + 8) × 3 = 33, then 33 × 9·8696 = 325·6968 square inches, the surface.

Or thus.

mn × 3·1416 = the circumference *m*O*n*V, or length of the ring, = ($\frac{1}{2}$ AC + CD + $\frac{1}{2}$ DB) × 3·1416 = (3 + 8) × 3·1416 = 34·5576.

Now AC^2 × ·7854 = 9 × ·7854 = 7·0686, area of a section of the ring; hence 7·0686 × 34·5576 = 244·27385136 inches, the solidity measured as a cylinder.

For the Surface.

AC × 3·1416 = 3 × 3·1416 = 9·4248, circumference of the section; hence 34·5576 × 9·4248 = 325·69846848 inches, the surface measured as a cylinder.

2. What is the solidity of the ring of a ship's anchor, the inner diameter being 25 inches, and the thickness of the metal 6·5 inches?

Ans. 3283·8 inches.

3. A jeweller had a cylindrical ring made for a sign, the inner diameter being 18 inches, and the thickness of the ring 4 inches: what was the expense of gilding it at one penny per square inch?

Ans. 868·5248 pence = £3 : 12*s.* : 4$\frac{1}{2}$*d.*

OF

OF THE FIVE REGULAR BODIES.

A **REGULAR BODY** is a solid contained under a certain number of similar and equal plane figures.

The whole number of regular bodies, which can possibly be formed, are five; for only three kinds of equilateral and equiangular plane figures, joined together, can make a solid angle *.

1. The *Tetraedron*, or equilateral pyramid, which has four triangular faces.
2. The *Octaedron*, which has eight triangular faces.
3. The *Icosaedron*, which has twenty triangular faces.
4. The *Dodecaedron*, which has twelve pentagonal faces.
5. The *Hexaedron*, or cube, which has six square faces.

A TABLE, *shewing the Solidity and Superficies of the* FIVE REGULAR BODIES, *the Length of a Side in each being one* †.

No. of Sides.	Names.	Solidities.	Surfaces.
4	Tetraedron.	·1178511	1.7320508
8	Octaedron.	·4714045	3·4641016
20	Icosaedron.	2·1816950	8·6602540
12	Dodecaedron.	7·6631189	20·6457288
6	Hexaedron.	1·0000000	6·0000000

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* *Keith's Geom.* 22 of X.

† The methods of calculating such a table as this may be found in Dr. *Hutton's* large Treatise on Mensuration, and in *Emerson's Geometry*.

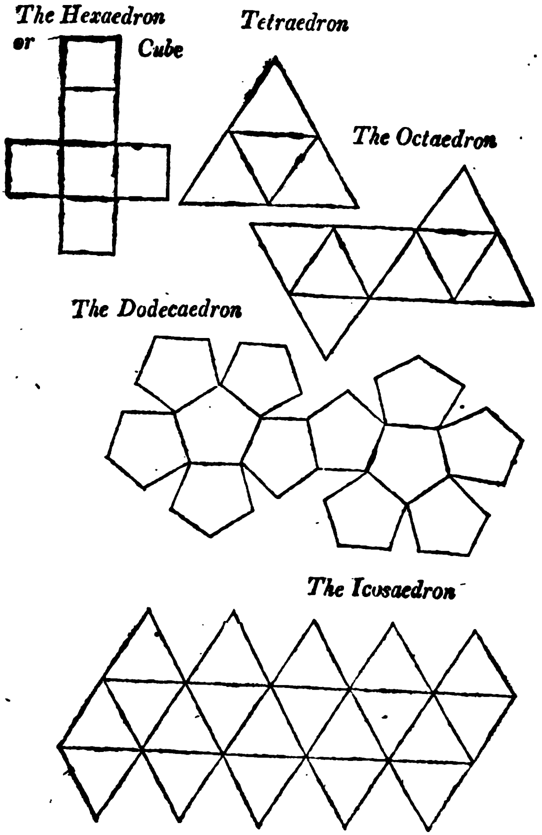
PRO-

PROBLEM XXIX.

To find the Solidity and Superficies of any regular Body.

Rule 1. Multiply the cube of the length of a side of the body by the tabular solidity, and the product will give the solidity of the body.

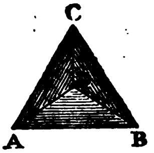
2. Multiply the square of the length of a side of the body, by the tabular superficies, and the product will give the superficies of the body.



Note. If the preceding figures be exactly drawn on pasteboard, and the lines be cut half through, so that the parts may be turned up and glued together, they will represent the five regular bodies. These bodies may all be inscribed in a sphere, so that each angle may touch its internal surface.

Example 1. The side A B, B C, B D, &c. of a *Tetraedron*, or equilateral trangular pyramid, is 12, what are its solidity and superficies.

$12^3 \times {\cdot}1178511 = 1728 \times {\cdot}1178511 = 203{\cdot}6467$, the solidity. And $12^2 \times 1{\cdot}7320508 = 144 \times 1{\cdot}7320508 = 249{\cdot}4153152$, the superficies.

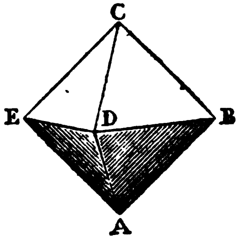


2. Each side of a tetraedron is 3, required its surface and solidity.

Ans. Surface 15·58845, solidity 3·18198.

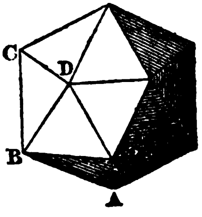
3. Each side A B, B D, B C, &c. of an Octaedron is 12 inches, required the solidity and superficies.

Ans.
Solidity=814·586976 in.
Superficies=498·8306304 in.

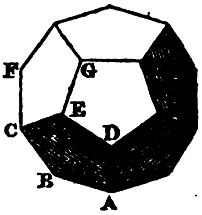


4. Each side A B, B D, B C, &c. of an Icosaedron is 12 inches, what are the solidity and superficies.

Ans.
Solidity=3769·96896 inches.
Superficies=1247·076576 inches.



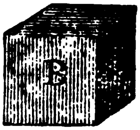
5. Each



5. Each side A B, B C, C E, &c. of a Dodecaedron is 12 inches, what are the solidity and superficies?

Ans.

Solidity=13241·8694592 in.
Superficies= 2972·9849472 in.



6. Let B be a Hexaedron or cube, whose side is 12 inches, required the solidity and superficies?

Ans. Solidity=1728 cubic inches.
Superficies = 864 square inches.

OF THE SPECIFIC GRAVITY OF SOLIDS.

1. The specific gravity of a body is the relation which the weight of a *given magnitude* of that body, has to the weight of an *equal magnitude* of a body of another kind.

2. A body immersed in a fluid, loses as much weight as an equal bulk of the fluid weighs. If the body be of equal density with the fluid, it loses all its weight, and requires no force but the fluid to sustain it. If it be heavier, its weight in the fluid will be the difference between its own weight and the weight of the same bulk of the fluid; and it will require a force to sustain it just equal to this difference. If it be lighter than the fluid, it will require a force equal to the difference between its own weight and the weight of the same bulk of the fluid, to keep it from rising in the fluid.

3. In the comparison of the weights of bodies, it is convenient to consider some one as a standard, with which all the others may be compared; and as rain water is nearly alike in all places, it is generally taken as the standard.

By repeated experiments, it has been found that a cubic foot of rain water weighs 62½ pounds avoirdupois, or 1000 ounces; hence the weight of a cubic inch is ·03616898148 pound. If therefore the specific gravity of any body be multiplied, by ·00003616898, the product will be the weight of a cubic inch of that body in pounds avoirdupois; and because 144 pounds avoirdupois are exactly equal to 175 pounds troy; if this weight be multiplied by 175, and the product be divided by 144, the quotient will give the weight of a cubic inch in pounds troy.

PROBLEM XXX.

To find the specific Gravity of a Body.

RULES.

Case 1. *When the Body is heavier than Water.*

Weigh it both in water and out of water, the difference will be the weight lost in water.

Then, the weight lost in water, is to the absolute weight of the body; as the specific gravity of water, is to the specific gravity of the body.

Case 2. *When the Body is lighter than Water, so that it will not sink.*

Fix another body to it, which is heavier than water, so that both may sink together, as a compound mass.

Weigh the heavier body and the compound mass separately, both in water and out of it, and find how much each loses in water, by subtracting its weight in water from its weight in air.

Then, the difference of these remainders, is to the weight of the lighter body in air; as the specific gravity of water, is to the specific gravity of the lighter body.

CASE 3. *For a Fluid of any Kind.*

Take part of a body of known specific gravity; weigh it both in and out of the fluid, and find the loss of weight by taking the difference of the two.

Then, the whole, or absolute weight, is to the loss of weight; as the specific gravity of the solid, is to the specific gravity of the fluid.

Examples to Case 1.

1. A piece of metal weighed 83·1886 pounds out of water, and in water only 79·5717 pounds, required its specific gravity; that of water being 1000.

83·1886 — 79·5717 = 3·6169 pounds, weight lost in water.

Then 3·6169 : 83·1886 :: 1000 : 23000 the specific gravity, or the weight of a cubic foot of the metal in ounces; hence, from the following Table, the metal appears to be *Platina.*

2. If a piece of stone weigh 108½ pounds out of water, and 72·3 pounds in water, what is its specific gravity? that of water being 1000. *Ans.* 2997 = 3000 nearly, the specific gravity of *granite.*

Examples to Case 2.

3. Suppose a piece of elm weighs 15 pounds in air; and that a piece of copper, which weighs 18 pounds in air, and 16 pounds in water, is affixed to it, if the compound weighs six pounds in water; required the specific gravity of the elm.

18 — 16 = 2 pounds, loss of weight in the copper, (15 + 18) — 6 = 27 pounds, loss of weight in the compound.

Then 27 — 2 : 15 pounds :: 1000 : 600, the specific gravity of the elm.

4. A piece of cork weighing 20 pounds in air, had a piece of granite fixed to it, that weighed 120 pounds in air, and 80 pounds in water; the compound mass

weighed 16⅔ pounds in water, what was the specific gravity of the cork?

Ans. 240 the specific gravity of the cork.

Examples to Case 3.

5. A piece of cast iron weighed 259·1 ounces in a fluid, and 298·1 ounces out of it; what was the specific gravity of the fluid? the specific gravity of cast iron being 7645.

298·1 — 259·1 = 39 loss of weight in the iron.

298·1 ounces : 39 :: 7645 : 1000, the specific gravity of the fluid, hence the fluid is common water.

6. A piece of lignum vitæ weighed 42¾ ounces in a fluid, and 166⅝ ounces out of it: what was the specific gravity of the fluid? that of lignum vitæ being 1333.

Ans. 991 the specific gravity of the fluid, hence the fluid is liquid turpentine.

A Table of the Specific Gravities of Bodies, and the Weight of a cubic Foot of each in Ounces Avoirdupois.

Platina, pure, hammered	23000	Nitre	1900
Gold, pure, hammered	19350	Ivory	1825
Standard gold	17724	Brimstone	1850
Quicksilver	13568	Sand	1520
Lead, cast	11325	Lignum vitæ	1333
Silver, pure, cast	10474	Newcastle coal	1270
Silver, pure, hammered	10510	Staffordshire coal	1240
Silver, standard	10351	Box wood	1030
Copper, pure, hammered	9000	Ebony	1177
Copper, cast	8830	Mahogany	1063
Gun metal	8784	Sea water	1030
Brass, cast	8396	Common water	1000
Steel	7816	Dry oak	932
Bar iron	7788	Liquid turpentine	991
Cast iron	7645	Gunpowder, shaken	930
Tin	7320	Ditto, in loose heap	829
Zinc, pure	6861	Logwood, dry	913
Zinc, hammered	7190	Beech, ditto	852
Granite	3000	Ash, ditto	845
Flint glass	2922	Apple tree, ditto	793
White glass	2892	Plum tree, ditto	755
Bottle glass	2732	Maple, ditto	755
Common green glass	2642	Cherry tree, ditto	715
White marble	2837	Walnut tree, ditto	671
Green and red marble	2742	Pear tree, ditto	661
Slate	2672	Yellow fir, ditto	657
Purbeck stone	2601	Elm, ditto	600
Portland stone	2570	Cypress, ditto	644
Mill stone	2484	Lime tree, ditto	604
Chinese porcelain	2385	White fir, ditto	569
Grindstone	2143	Cedar, ditto	561
Salt	2130	Willow, ditto	585
Clay	2160	Poplar, ditto	383
Brick	2000	Cork	240
Common earth	1984	Air	1·225

PRO-

PROBLEM XXXI.

The linear Dimensions, or Magnitude of a Body, and its specific Gravity, being given, to find its Weight.

Rule. One cubic foot, or 1728 cubic inches, is to the solidity of the body, in inches; as the tabular specific gravity of the body, is to its weight in avoirdupois ounces.

Example 1. What is the weight of a piece of dry oak, in the form of a parallelopipedon, whose length is 56 inches, breadth 18, and depth 12?

$56 \times 18 \times 12 = 12096$ cubic inches, the solidity.

Then 1728 : 12096 :: 932 : 6524 ounces = $407\frac{3}{4}$ pounds avoirdupois, the weight required.

2. What is the weight of a cast iron shot of $3\frac{1}{2}$ inches in diameter?

Ans. 99·32 ounces = 6·2 pounds.

3. Required the weight of a cast iron shell of 9 inches diameter, the metal being $1\frac{1}{2}$ inch thick?

Ans. 1188·367 ounces = 74·273 pounds.

4. Required the weight of one of the Portland keystones, to the middle arch of Westminster bridge: the diameter of the arch being 76 feet, the height of the keystone 5 feet, the upper chord of the front of the stone being 3 feet 4 inches, and the depth of the stone 4 feet.

Ans. The lower chord of the front of the stone is 3·127572 feet, (*vide Note, p.* 63); and as the chords and the arcs do not essentially differ, the stone may be measured as a prismoid, its content will be found to be 111644·4384 cubic inches; hence its weight = 166045·2585 ounces = 10377·8286 pounds = 4 tons, 12 cwt. 2 quarters, 17·8286 pounds.

5. Wanting to know the solidity and weight of an irregular piece of flint glass, and having no means of

weighing it, I put it into a cylindrical vessel, whose diameter was 32 inches, and then poured as much water into the vessel as just covered the glass; on taking the solid out of the vessel, I found the fall of the water to be 14 inches; what were the solidity and weight of the glass?

Ans. { Solidity = 11259·49 inches.
Weight = 19039·48 ounces.

PROBLEM XXXII.

The specific Gravity of a Body, and its Weight, being given, to find its Solidity.

Rule. The tabular specific gravity of the body, is to its weight, in ounces avoirdupois; as 1 cubic foot, or 1728 inches, is to the content in feet, or inches.

Example 1. What is the solidity of a block of white marble, that weighs 8 tons, 14 cwt.; and what will it come to at 8s. per solid foot.

First 8 tons, 14 cwt. = 19488 pounds = 311808 ounces.

2837 : 311808 :: 1 cubic foot : 109·9076 feet, the solidity; which, at 8s. per foot, amounts to £43 : 19 : 3·1296.

2. What is the diameter of a cast-iron shot, that weighs 42 pounds?

Ans. { Solidity = 151·892 cubic inches.
Diameter = 6·62, nearly.

3. How many cubic inches of gunpowder are contained in a pound avoirdupois? *Ans.* 29·7.

PROBLEM XXXIII.

To find the Quantities of two Ingredients in a given Compound.

Rule*. Find the solidity of the given compound (by Prob. XXXII), and the solidities of two equal

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* This Rule is derived from Alligation. Let c = the specific gravity of the compound, g = the greater of the other two specific gravities, l = the less, and w = the weight of the compound. Then the respective solidities, (by Prob. XXXII.) will be $\frac{w}{c}$, $\frac{w}{g}$, and $\frac{w}{l}$. Now, (by *Keith's Arithmetic*, Proposition 4th, *Alligation*) we have

$$\frac{w}{c}\left|\begin{array}{ll} \frac{w}{g} & \frac{w}{l}-\frac{w}{c} \quad \text{greater difference.} \\ \frac{w}{l} & \frac{w}{c}-\frac{w}{g} \quad \text{less difference.} \\ & \overline{\frac{w}{l}-\frac{w}{g}} \quad \text{Sum of the differences.} \end{array}\right.$$

Then, $\frac{w}{l}-\frac{w}{g} : w :: \frac{w}{l}-\frac{w}{c} : Q$, the quantity of the heavier ingredient, which is the Rule. Also, $\frac{w}{l}-\frac{w}{g} : w :: \frac{w}{c}-\frac{w}{g} : q$ the quantity of the lighter ingredient. But these proportions may be simplified by reduction; the first becomes $\frac{g-l}{lg} : w :: \frac{c-l}{lc} : Q$; or, $\frac{(c-l)\times g}{(g-l)\times c} \times w = Q$, or $(g-l)\times c : (c-l)\times g :: w : Q$, which is the second Rule. The second proportion, by reduction, becomes $\frac{g-l}{lg} : w :: \frac{g-c}{lg} : q$; or, $\frac{(g-c)\times l}{(g-l)\times c} \times w = q$, or $(g-l)\times c : (g-c)\times l :: w : q$, the quantity of the lighter ingredient. The same method may be applied to any number of ingredients.

weights, having the same specific gravities as each of the ingredients, which constitute the given compound.

Find the difference between the solidity of the compound, and each of these solidities; then, the sum of these differences, is to the weight of the compound; as the greater difference, is to the quantity of the heavier ingredient.

Or, which is the same,

From the specific gravity of the compound, subtract the less of the other two, and multiply the remainder by the greatest; also, from the greatest specific gravity, subtract the least, and multiply the remainder by the specific gravity of the compound.

Then, the second product is to the first, as the whole weight of the compound is to the weight of the ingredient, having the greatest specific gravity.

Example 1. The specific gravity of a compound of tin and copper is 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and that of copper 9000; also the weight of the mass being 112 pound.

8784 : 112 :: 1 : ·012750455 feet, the solidity of the compound.

9000 : 112 :: 1 : ·012444444 feet, solidity of the copper.

7320 : 112 :: 1 : ·015300546 feet, solidity of the tin.

Then ·012750455 — ·012444444 = ·000306011
And ·015300546 — ·012750455 = ·002550091

Sum = ·002856102

Lastly, ·002856102 : 112 pounds :: ·002550091 : 100 pounds, the weight of copper; hence the tin weighs 12 pounds.

Or thus:

(8784 — 7320) × 9000 = 1464 × 9000 = 13176000, first product.

(9000 — 7320) × 8784 = 1680 × 8784 = 14757120, second product.

Then 14757120 : 13176000 :: 112 pounds : 100 pounds, the weight of the copper, as before.

2. A vase, made of pure gold and silver, hammered, weighed 128 pounds, and its specific gravity was 16640; what quantity of each sort of these metals was in its composition? the specific gravity of the gold being 19350, and of the silver 10510.

Ans. { 103·2157 pounds of gold.
24·7843 pounds of silver.

3. It is reported that *Hiero*, King of *Sicily*, furnished a goldsmith with a quantity of gold, to make a crown. When it came home, he suspected that the goldsmith had used a greater alloy of silver than was necessary in the composition; and applied to *Archimedes*, a celebrated mathematician of *Syracuse*, to discover the fraud, without defacing the crown.

He procured a mass of gold, and another of silver, each exactly of the same weight with the crown: considering, that if the crown were of pure gold, it would be of equal bulk, and displace an equal quantity of water with the golden mass; and if of silver, it would be of equal bulk, and displace an equal quantity of water with the silver mass; but if a mixture of the two, it would displace an intermediate quantity of water.

Now, suppose that each of the three weighed 100 ounces; and that on immersing them severally in water, there were displaced 5 ounces of water by the golden mass, 9 ounces by the silver mass, and 6 ounces by the crown; what quantity of gold and of silver did the crown contain?

Ans. { 75 ounces of gold.
25 ounces of silver.

PROBLEM XXXIV.

To find how many Inches a floating Body of given Dimensions will sink in a Fluid.

RULE. Find the weight of the floating body from its solidity and specific gravity, (Prob. XXXI.) and that will be the weight of the fluid which it will displace.

Then, the specific gravity of the fluid, is to 1728 cubic inches; as the weight of the body, in ounces, is to the cubic inches immersed. The depth will be found from the given dimensions.

Example 1. A piece of dry oak, in the form of a parallelopipedon, whose length is 56 inches, breadth 18, and depth 12, is to be floated upon common smooth water, on its broadest side; how many inches will it sink? its specific gravity being 932.

Its weight, by Example 1, Prob. XXXI, is 6524 ounces, which is the weight of water displaced.

Then 1000 spec. of the water : 1728 :: 6524 : 11273·472 cubic inches of oak immersed. Consequently 11273·472 ÷ (56 × 18) = 11·184 inches the depth it will sink; if it be floated on its narrower side, it will sink 16·776 inches.

2. How many inches will a cubic foot of dry elm sink in common water, the specific gravity of elm being 600? *Ans.* 7·2 inches.

3. The length of a piece of yellow fir is 40 feet, each side of its square ends 3 feet, if it be floated in smooth sea water, to what depth will it sink? the specific gravity of yellow fir being 657, and of sea water 1030. *Ans.* 22·9631 inches.

PROBLEM XXXV.

To find what Weight may be attached to a floating Body, so that it may be just covered with the Fluid.

RULE. Multiply the cubic feet in the body by the difference between its specific gravity and that of the fluid, the product will be the weight in ounces avoirdupois, just sufficient to immerse it in the fluid.

Example 1. What weight, fixed to a piece of dry oak, 56 inches long, 18 inches broad, and 12 inches deep, will keep it from rising above the surface of a fresh water lake; the specific gravity of the water being 1000, and that of the oak 932?

First $56 \times 18 \times 12 = 12096$ cubic inches = 7 feet.

Then $7 \times (1000 - 932) = 7 \times 68 = 476$ ounces = 29 pounds, 12 ounces.

2. What weight will be just sufficient to immerse a cubic foot of cork in sea water? *Ans.* 49⅜ pounds.

3. A sailor had half an anker of brandy, the specific gravity of the liquor was 927, the cask was oak, and contained 216 cubic inches, and its specific gravity was 932: to secure his prize from the Custom-house officers, he fixed just as much lead to the cask as would keep it under water, and then threw it into the sea; what weight of lead was necessary for his purpose?

Ans. The cask and brandy contained 1371 cubic inches, the weight of sea water of an equal bulk was 817·20486 ounces avoirdupois; the cask weighed 116·5 ounces, the brandy 619·609375, both together weighed 736·19375 ounces, then $817{\cdot}20486 - 736{\cdot}19375 = 81{\cdot}01111$ ounces. Now the difference between the specific gravity of lead and of sea water is to this remainder, as the specific gravity of lead is to its weight in ounces, which will be found to be 89·09495 ounces avoirdupois, or 5 pounds 9 ounces.

PROBLEM XXXVI.

To find the Solidity of a Body, lighter than a Fluid, which will be sufficient to prevent a Body, much heavier than the Fluid, from sinking.

Rule. Find the solidity of the body to be floated, from its weight and specific gravity, (by Prob. XXXII.) and the weight of an equal bulk of the fluid, (by Prob. XXXI.)

Then, the difference between the specific gravity of the fluid, and that of the body lighter than the fluid, is to the difference between the weight of the body to be floated, and the weight of an equal bulk of the fluid; as 1728 is to the solidity of the lighter body, in cubic inches.

Example 1. How many solid feet of yellow fir, whose specific gravity is 657, will be sufficient to keep a brass cannon, weighing 56 cwt. afloat at sea? the specific gravity of brass being 8396, and of sea water 1030.

First, 56 cwt. = 100352 ounces, weight of the body to be floated; and 8396 : 100352 :: 1728 : 20653·675 cubic inches in the cannon.

Then, 1728 : 20653·675 :: 1030 : 12310·9289, the weight of sea water equal in bulk to that of the cannon.

Now, 1030 — 657 : 100352 — 12310·9289 :: 1728 : 407868·5545 cubic inches = 236·035 feet.

2. How many cubic inches of cork will be sufficient to keep 49¼ pounds of lead afloat at sea? the specific gravity of lead being 11325, that of cork 240, and of sea water 1030.

Ans. 1570·84 cubic inches = 1 foot nearly.

3. How many cubic inches of white fir will be sufficient to keep a piece of lignum vitæ, that weighs 146 pounds, from sinking in common water? the specific gravity of lignum vitæ being 1333, that of white fir 569, and of common water 1000.

1 *Ans.*

Ans. 146·229 cubic inches being equal to a piece of fir 3 inches square, and 16¼ inches long; and a less piece of fir than this would save a man from being drowned*, as long as he had strength and spirits to keep his hold.

WEIGHT AND DIMENSIONS OF BALLS† AND SHELLS.

PROBLEM XXXVII.

The Diameter of an Iron Shot being given, to find its Weight.

RULE. As 64 is to 9 pounds; so is the cube of the diameter, to the weight.

Example 1. What is the weight of an iron ball, whose diameter is 3½ inches?

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* Mr. *John Robertson*, late librarian to the Royal Society, and author of a Treatise on Navigation, found by experiment, that the specific gravity of *living* men is something less than that of common water. The average specific gravity of *ten* men of different sizes, from 6 feet 2 inches, to 5 feet 3 inches, and weighing from 161 pounds, to 132 pounds, was 891, the average weight of the men was 146 pounds. See the Philosophical Transactions, vol. l. art. 5.

† These may be found by their specific gravities; but they may be determined more simply, by having the weight and dimensions of one given; because the weights are as the cubes of the diameters. *Holliday*, in his *Gunnery*, p. 115, second edition, has given a table of the weights of iron and leaden balls, from 2 to 7 inches in diameter. The weight of an iron ball of 4 inches in diameter, is 9 pounds avoirdupois; and that of a leaden ball of 4½ inches in diameter, is 17 pounds.

The methods of piling balls and shells, finding the quantity of powder a shell, hollow cylinder, or box, will hold, form no essential part of mensuration, and are useless, except in an *arsenal*, for which reason they are omitted in this Treatise. The subject is amply treated of by *Holliday*.

$3{\cdot}5 \times 3{\cdot}5 \times 3{\cdot}5 = 42{\cdot}875$, cube of the diameter.

Then, 64 : 9 pounds :: 42·875 : 6·0293 pounds, the weight.

2. The diameter of an iron shot is 7 inches, required its weight. *Ans.* 48·234 pounds.

PROBLEM XXXVIII.

The Diameter of a Leaden Ball being given, to find its Weight.

Rule. As 289 is to 64; so is the cube of the diameter, to the weight.

Example 1. What is the weight of a leaden ball of $3\frac{1}{4}$ inches in diameter?

$3{\cdot}25 \times 3{\cdot}25 \times 3{\cdot}25 = 34{\cdot}328125$, cube of the diameter.

Then 289 : 64 :: 34·328125 : 7·6 pounds, the weight required.

2. The diameter of a leaden ball is $5\frac{1}{2}$ inches, required its weight. *Ans.* 42·1 pounds.

PROBLEM XXXIX.

The Weight of an Iron Ball being given, to find its Diameter.

Rule. As 9 is to 64, so is the weight to the cube of the diameter; the cube-root will be the diameter.

Or, the diameter is equal to $\frac{4}{3}$ of the cube-root of 3 times the weight.

Example 1. The weight of an iron ball is 42 pounds, what is its diameter?

$$9 : 64 :: 42 : \frac{42 \times 64}{9} = \frac{64}{27} \times 126.$$

Then $\sqrt[3]{\frac{64}{27} \times 126} = \frac{4}{3} \sqrt[3]{126} = 6\cdot6844$ inches.

2. An iron ball weighs 24 pounds, what is its diameter?

Ans. $\frac{4}{3} \sqrt[3]{72} = 5\cdot54689$ pounds.

PROBLEM XL.

The Weight of a Leaden Ball being given, to find its Diameter.

Rule. As 64 is 289, so is the weight to the cube of the diameter; the cube-root will be the diameter.

Or, multiply the weight by 289, and one-fourth of the cube-root will be the diameter.

Example 1. The weight of a leaden ball is $42\frac{1}{10}$ pounds, what is its diameter?

$$64 : 289 :: 42\cdot1 : \frac{42\cdot1 \times 289}{64}.$$

Then $\sqrt[3]{\frac{42\cdot1 \times 289}{64}} = \sqrt[3]{\frac{1}{64} \times 42\cdot1 \times 289}$ $= \frac{1}{4} \sqrt[3]{42\cdot1 \times 289} = \frac{1}{4} \sqrt[3]{12166\cdot9} = 5\cdot7499 =$ $5\frac{3}{4}$ inches.

2. The weight of a leaden ball is 8 pounds, what is its diameter? *Ans.* $\frac{1}{2} \sqrt[3]{289} = 3\cdot305744$ inches.

PROBLEM XLI.

The external and internal Diameters of an Iron Shell being given, to find its Weight.

Rule *. From the square of the sum of the two diameters, take their product, multiply the remainder by the difference of the diameters, and $\frac{9}{64}$ of the result will be the weight.

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* *Demonstration.* Let $D =$ the external diameter, and $d =$ the internal diameter, then (by rule Prob. XXXVII.) $\frac{9}{64} D^3 - \frac{9}{64} d^3$ will be the weight of metal, $= \frac{9}{64} \times (D^3 - d^3) = \frac{9}{64} \times (D^2 + Dd + d^2) \times (D - d) = \frac{9}{64} \times (\overline{D + d}^2 - Dd) \times (D - d)$, which is the rule.

Example 1. What is the weight of a 13 inch iron bomb shell, the metal being 2 inches thick?

First, $13 - 4 = 9$, the internal diameter.

Then $\overline{13 + 9}^2 - (13 \times 9) = 484 - 117 = 367$ remainder, and $367 \times (13 - 9) = 1468$; which, multiplied by 9, and divided by 64, gives 206·437 pounds, *Answer*.

2. The external and internal diameters of an iron shell are 9·8 and 7 inches, what is the weight?

Ans. 84·12 pounds.

THE

COMPLETE MEASURER.

The Description of the CARPENTERS RULE, *Mensuration of* TIMBER, *and of* ARTIFICERS WORK.

1. OF THE CARPENTERS RULE.

THIS rule is generally used in measuring of timber, and artificers works; not only in taking dimensions, but in casting up the contents of such work.

It consists of two equal pieces of box, each one foot in length, connected together by a folding joint; in one of these equal pieces there is a slider, and four lines marked at the right-hand A, B, C, D; two of these lines, B, C, are upon the slider, and the other two, A, D, upon the rule.

Three of these lines, viz. A, B, C, are called double lines, because they proceed from 1 to 10 twice over; these three lines are all exactly alike, both in numbers and division. They are numbered from the left-hand towards the right 1, 2, 3, 4, 5, 6, 7, 8, 9, 1 which stands in the middle; the numbers then go on 2, 3, 4, 5, 6, 7, 8, 9, 10 which stands at the right hand end of the rule.

These numbers have no determinate value of their own, but depend upon the value you set on the unit at the left-hand of this part of the rule; thus if you call it 1, the 1 in the middle will be 10, the other figures which follow will be 20, 30, &c. and the 10 at the right-hand end will be 100. If you call the first, or left-hand unit 10, the middle 1 will be 100, and the following figures will be 200, 300, 400, &c. and the 10 at the right-hand end will be 1000. Or, if you call the first, or left-hand unit, 100, the middle 1 will be 1000, and the following figures 2000, 3000, 4000, &c. and the 10 at the right-hand 10,000. Lastly, according as you alter, or number, the large divisions, so you must alter the small divisions proportionably.

The fourth line D, is a single line, proceeding from 4 to 40; it is also called the *Girt-Line*, from its use in casting up the contents of trees and timber. Upon it are marked W G at 17·15, and A G at 18·95, the wine and ale gauge points, to make it serve the purpose of a gauging rule.

The use of the double lines A and B, is for working the rule of proportion, and finding the areas of plane figures. And the use of the *girt-line* D, and the other double line C, is for measuring of timber. On the other part of this side of the rule, there is a table of the value of a load, or 50 cubic feet, of timber, at all prices, from 6 pence to 24 pence, or two shillings, per foot.

On the other side of the rule are several plane scales divided into 12th parts, marked *inch*, ¾, ½, ¼, &c. signifying, that the inch, ¾ inch, &c. are each divided into 12 parts. These scales are useful for planning dimensions that are taken in feet and inches. The edge of the ruler is divided into inches, and each of these inches into eight parts, representing half inches, quarter inches, and half quarters.

In this description we have supposed the rule to be folded; let it now be opened, and slide out the slider, you will find the back part of it divided like the edge of the rule, so that all together will measure 1 yard or 3 feet in length.

Some rules have other scales and tables upon them; as a table of board measure, one of timber measure; a line for shewing what length for any breadth will make a foot square; also a line shewing what length for any thickness will make a solid foot: the former line serves to complete the table of board measure, and the latter the table of timber measure.

The thickness of the rule is generally about a quarter of an inch; this face is divided into inches and tenths, and numbered, when the rule is opened, from the right-hand towards the left, 10, 20, 30, 40, &c. to 100, which falls upon the joint; the other half is numbered in the same manner, and the same way. This scale serves for taking dimensions in feet, tenths, and hundredths of a foot, which is the most commodious way of taking dimensions, when the contents are cast up decimally.

The Use of the SLIDING RULE.

PROBLEM I.

To multiply Numbers together, as 12 *and* 16.

Set 1 on B, to the multiplier (12) on A; then against the multiplicand (16) on B, stands the product (192) on A.

2. *Find the Product* 35 *and* 19.

Set 1 on B, to the multiplicand (35) on A; then because 19 on B runs beyond the rule, I look for 1·9 on B, and against it on A, I find 66·5: but the real multiplier was divided by 10, therefore the product 66·5 must be multiplied by 10, which is done by taking away the decimal point, so the product is 665.

PROBLEM II.

To divide one Number by another, as 360 *by* 12.

Set the divisor (12) on A, to 1 on B; then against the dividend (360) on A, stands the quotient (30) on B.

2. *Divide*

2. *Divide 7680 by 24.*

Set the divisor (24) on A, to 1 on B; then because 7680 is not contained on A, I look for 768 on A, and against it I find 32 on B, the quotient; but because one-tenth of the dividend was taken to make it fall within the compass of the scale A, the quotient must be multiplied by 10, which gives 320.

PROBLEM III.

To square any Number, as 25.

Set 1 upon C to 10 upon D; then observe, that if you call the 10 upon D, 1, the 1 on C will be 1; if you call the 10 on D, 10, then the 1 on C will be 100; if you call the 10 on D, 100, then the 1 on C will be 10000, &c. This being well understood, you will observe that against every number on D, stands its square on C.

Thus against 25 stands 625
against 30 stands 900
against 35 stands 1225
against 40 stands 1600

Reckoning the 10 on D, to be 10.

PROBLEM IV.

To extract the Square Root of a Number.

Fix the slider exactly as in the preceding Problem, and estimate the value of the lines D and C in the same manner; then against every number found on C stands its square root on D.

PRO-

PROBLEM V.

To find a mean Proportional between two given Numbers, as 9 and 25.

Set the one number (9) on C, to the same (9) on D; then against 25 on C, stands 15 on D, the mean proportional sought.

For, 9 : 15 :: 15 : 25.

2. What is the mean proportional between 29 and 430?

Set 29 on C, to 29 on D; this being done, you will find that 430 on C will either fall beyond the scale D, or it will not be contained on C. Therefore take the 100th part of it, and look for 4·3 on C, and against it on D stands 11·2, which multiply by 10, and 112 is the mean proportional required.

PROBLEM VI.

To find a fourth Proportional to three Numbers; or, to perform the Rule of Three.

Suppose it were required to find a fourth proportional to 12, 28, and 57. Set the first term (12) on B to the second (28) on A; then against the third term (57) on B stands the fourth (133) on A.

If either of the middle numbers fall beyond the line, take one-tenth part of that number, and increase the answer 10 times.

Note. The finding a third proportional between two numbers is exactly the same, for in such cases the second number is repeated to form the third: thus the third proportion to 12 and 28 is 65·33, and is thus found 12 : 28 :: 28 : 65·33, viz. set 12 on B, to 28 on A; then against 28 on B stands 65·3 on A.

II. OF TIMBER MEASURE.

PROBLEM I.

To find the superficial content of a board or plank.

RULE. Multiply the length by the breadth.

Note. When the board is broader at one end than the other, add the breadth of the two ends together, and take half the sum for a mean breadth.

By the Carpenters Rule.

Set 12 on B to the breadth in inches on A; then against the length, in feet, on B, you will find the superficies on A, in feet.

Example 1. If a board be 10 feet 6 inches long, and 1 foot 3 inches broad; how many feet are contained in it?

By Decimals.	*By Duodecimals.*
Feet.	*Feet. In.*
10·5	10 : 6
1·25	1 : 3
525	10 : 6
210	2 : 7 : 6
105	
	13 : 1 : 6 *Ans.*
13·125 feet, *Ans.*	

By the Carpenters Rule.

As 12 *on* B : 15 *on* A :: 105 *on* B : $13\frac{1}{8}$ *on* A.

2. What is the value of a plank, whose length is 8 feet 6 inches, and breadth throughout 1 foot 3 inches; at $2\frac{1}{2}$d. per foot? *Ans.* 2 shill. $2\frac{1}{2}$d.

3. Required the superficies of a board, whose mean breadth is 1 foot 2 inches, and length 12 feet 6 inches? *Ans.* 14 feet 7 inches.

4. Having occasion to measure an irregular mahogany plank of 14 feet in length, I found it necessary to measure several breadths at equal distances from each other, *viz.* at every two feet. The breadth of the less end was 6 inches, and that of the greater end 1 foot; the intermediate breadths were 1 foot, 1 foot 6 inches, 2 feet, 2 feet 1 foot, and 2 feet: How many square feet were contained in the plank?

Ans. 19¼ feet;—mean breadth 1⅜ feet, found by dividing the sum of the breadths by their number.

5. Required the value of 5 oaken planks at 3d. per foot, each of them being 17½ feet long; and their several breadths as follow; *viz.* two of 13½ inches in the middle, one of 14¼ inches in the middle, and the two remaining ones, each 18 inches at the broader end, and 11¼ inches at the narrower? *Ans.* £1 5 8¼

PROBLEM II.

Given the breadth of a rectangular plank in inches, to find how much in length will make a foot, or any other assigned quantity.

Rule. Divide 144, or the area to be cut off, by the breadth in inches, and the quotient will be the length in inches.

Note. To answer the purpose of the above rule, some carpenters rules have a little table upon them, in the following form, called *A table of board measure.*

0	0	0	0	5	0	8½	6	Inches
12	6	4	3	2	2	1	1	Feet
1	2	3	4	5	6	7	8	Breadth

This table shews, that if the breadth be 1 inch, the length must be 12 feet; if 2 inches, the length is 6 feet; if 5 inches broad, the length is 2 feet 5 inches, &c.

If the breadth be not contained in the table on the rule, shut the rule, and look for the breadth in the line of board measure, which runs along the rule from the table of board measure, and over against it on the opposite side, in the scale of inches, is the length required. Thus if the breadth be 9 inches, you will find the length against 15 inches: if the breadth be 11 inches, you will find the length a little above 13 inches, &c.

Example 1. If a board be 19 inches broad; how many inches in length will make a foot?

Ans. 7·58 inches.

2. From a mahogany plank 26 inches broad, a yard and a half is to be cut off; at what distance from the end must the line be struck?

Ans. 74·7692 inches, or 6·23 feet.

PROBLEM III.

To find the solid Content of squared, or four sided Timber.

Rule 1 *. Take half the sum of the breadth and depth in the middle (*viz. the Quarter Girt*), square this half sum and multiply it by the length, the product will be the content.

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* The *first* rule is the *customary method*, and is certainly *incorrect* when the mean breadth and depth differ materially from each other, and the timber does *not* taper: in such cases the *second* rule should be used, as being *correct*. But this second rule is likewise subject to error; for, if the timber taper considerably, and the breadth and depth be nearly equal, it is more erroneous than the first. The *third* rule is perfectly correct whether the timber be of equal breadth and depth throughout, or taper regularly. Let B and b be the breadths of the two ends, D and d the depths, and L = the length; then BD = area of the greater end and bd that of the less, also $\frac{B+b}{2} \times \frac{D+d}{2} \times 4 =$

By the Carpenters Rule.

As 12 *on* D : length *on* C :: Quarter girt *on* D : solidity *on* C.

RULE II. Multiply the breadth in the middle by the depth in the middle, and that product by the length for the solidity.

Note. If the tree taper regularly from one end to the other, half the sum of the breadths of the two ends will be the breadth in the middle, and half the sum of the depths of the two ends will be the depth in the middle.

RULE III. Multiply the sum of the breadths of the two ends by the sum of the depths, to which add the product of the breadth and depth of each end; one-sixth of this sum multiplied by the length, will give the *correct* solidity, of any piece of squared timber tapering regularly.

Example 1. If a piece of squared timber be 2 feet 9 inches broad, 1 foot 7 inches deep, and 16 feet 9 inches long, how many solid feet are contained therein?

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4 = four times the area of the middle section = $(B+b)\times(D+d)$, hence $\frac{1}{6}(BD + (\overline{B+b}\times\overline{D+d}) + bd)\times L =$ the solidity, which is the *third* rule. See the Note to Prob. X. p. 114. $\frac{\left.\overline{(B+b)\times(D+d)}\right|^{2}}{4}\times L =$ the solidity by the *first* rule; and $\frac{B+b}{2}\times\frac{D+d}{2}\times L =$ the solidity by the *second* rule

By Rule I.

Ft. In.
2 : 9
1 : 7
———
2)4 : 4 Sum
———
2 : 2 *quarter girt*
2 : 2
———
4 : 4
4 : 4
———
4 : 8 : 4 square of ditto
16 : 9 length
———
75 : 1 : 4
3 : 6 : 3
———
78 : 7 : 7 solid content.

By Rule II.

Ft. In.
2 : 9 breadth
1 : 7 depth
———
2 : 9
1 : 7 : 3
———
4 : 4 : 3 product
16 : 9 length
———
69 : 8
3 : 3 : 2 : 3
———
72 : 11 : 2 : 3 solid cont.

Note. The answer by rule II. is *correct* in this example.

By the Carpenters rule.

As 12 *on* D : 16¾ *on* C :: 26 *on* D : 78½ *on* C.

2. A squared piece of timber is 15 inches broad, 15 inches deep, and 18 feet long, what is its solidity?

Ans. { 28⅛ feet by rule 1. / The same by rule 2. } *correct.*

3. If a piece of squared timber be 25 inches square at the greater end, 9 inches square at the less, and 20 feet long, how many feet of timber are contained therein?

Ans. { 40·138 feet by rule 1, the quarter girt being 17 inches.
The same by rule 2.
43·10185 feet by rule 3, *correct.*

4. If a piece of squared timber be 32 inches broad and 20 inches deep at the greater end; 10 inches broad and 6 inches deep at the less end, and 18 feet in length, how many feet of timber are contained therein?

Ans. { 36·125 feet by rule 1, quarter girt 17 inches.
34·125 feet by rule 2.
37·333 feet by rule 3. *Correct.*

7. A piece of squared timber is 30 inches broad and 18 inches deep at the greater end; 15 inches broad and 7 inches deep at the less end, and 30 feet long, how many square feet of timber are contained in it?

Ans. { 63·8 feet by rule 1, quarter girt 175 inches.
58·59375 feet by rule 2.
61·4583 feet by rule 3. *Correct.*

PROBLEM IV.

To find how much in length will make a solid foot, or any other assigned quantity, of squared timber, of equal dimensions from end to end.

Rule. Divide 1728 the solid inches in a foot, or the solidity to be cut off, by the area of the end in inches, and the quotient will be the length in inches.

Note. To answer the purpose of the above rule, some carpenters rules have a little table upon them, in the following form, called a *table of timber measure.*

0	0	0	0	9	0	11	3	9	inches
144	36	16	9	5	4	2	2	1	feet
1	2	3	4	5	6	7	8	9	side of the square,

This table shews, that if the side of the square be 1 inch, the length must be 144 feet; if 2 inches be the side of the square, the length must be 36 feet, to make a solid foot.

If the side of the square be not in the little table, look for it in the line of timber measure running along the rule from the table, and against it, in the line of inches, is the length required. Thus, if the side of the square be 16 inches, you will find the length to be 6 inches and 7 tenths.

Example 1. If a piece of timber be 18 inches square, how much in length will make a solid foot? *Ans.* $5\frac{1}{3}$ inches.

2. If a piece of timber be 22 inches broad, and 15 inches deep, how much in length will make a solid foot? *Ans.* 5·23 inches.

PROBLEM V.

To find the solidity of round or unsquared timber.

RULE I.

Gird the timber round the middle with a string; one-fourth part of this girt squared and multiplied by the length will give the solidity.

Note. If the circumference be taken in inches, and the length in feet, divide the last product by 144.

RULE II, *by the Table.*

Multiply the area corresponding to the quarter-girt in inches, by the length of the piece of timber in feet, and the product will be the solidity.

Note. If the quarter girt exceed the table, take half of it, and four times the content thus found will be the answer.

RULE III, *by the Carpenters Rule.*

Measure the circumference of the piece of timber in the middle, and take a quarter of it in inches, call this the girt.

Then set 12 *on* D to the length in feet *on* C, and against the girt in inches *on* D, you will find the content in feet *on* C.

If a tree be very irregular, or if it be much smaller at one end than at the other, divide it into several lengths, in such a manner that each length may be as nearly cylindrical as possible, and measure each part separately.

The girt of a tree is generally taken with a small piece of whipcord, having a knot upon it, the girt is doubled twice, or divided into four equal parts, and then applied to a carpenters rule, the number of inches contained in it, is the quarter girt. Out of this quarter girt an allowance is made for bark, *viz.* for ash, beech, and lime, about half an inch, for oak and elm from one inch to two inches according to the thickness of the bark, which in a great measure depends on the age of the tree. The deduction must be taken from the *whole* quarter girt, whatever may be its extent.

The *buyer* is allowed to take the girt any where between the greater end and the middle of the tree if it taper regularly, and the two ends do not very materially differ from each other; but when there is a great difference between the two ends, the *seller* will have the tree measured in two or three separate lengths, and the girts taken at the middle of each length.

All branches where the quarter girt is not less than 6 inches are reckoned as timber; and any part of the trunk less than 24 inches in circumference is not considered as timber, and is generally cut off.

A TABLE *for measuring* TIMBER.

Quarter GIRT.	AREA.	*Quarter* GIRT.	AREA.	*Quarter* GIRT.	AREA.
Inches.	*Feet.*	*Inches.*	*Feet.*	*Inches.*	*Feet.*
6	·250	12	1·000	18	2·250
6¼	·272	12¼	1·042	18½	2·376
6½	·294	12½	1·085	19	2·506
6¾	·317	12¾	1·129	19½	2·640
7	·340	13	1·174	20	2·777
7¼	·364	13¼	1·219	20½	2·917
7½	·390	13½	1·265	21	3·062
7¾	·417	13¾	1·313	21½	3·209
8	·444	14	1·361	22	3·362
8¼	·472	14¼	1·410	22½	3·516
8½	·501	14½	1·460	23	3·673
8¾	·531	14¾	1·511	23½	3·835
9	·562	15	1·562	24	4·000
9¼	·594	15¼	1·615	24½	4·168
9½	·626	15½	1·668	25	4·340
9¾	·659	15¾	1·722	25½	4·516
10	·694	16	1·777	26	4·694
10¼	·730	16¼	1·833	26½	4·876
10½	·766	16½	1·890	27	5·062
10¾	·803	16¾	1·948	27½	5·252
11	·840	17	2·006	28	5·444
11¼	·878	17¼	2·066	28½	5·640
11½	·918	17½	2·126	29	5·840
11¾	·959	17¾	2·187	29½	6·044
				30	6·250

Example 1. If a piece of round timber be 18 feet long, and the quarter girt 24 inches, how many feet of timber are contained therein?

24 quarter girt
24

96
48

576 square
18

4608
576

144)10368(72 feet.
1008

288
288

By the Table.

Against 24 stands 4·00
Length 18

Product - 72·00

Ans. 72 feet.

By the Carpenters Rule.

12 *on* D : 18 *on* C : : 24 *on* D : 72 *on* C.

2. If a piece of round timber be 20 feet long, and the quarter girt be 21½ inches, what is the solidity? *Ans.* 64·2 feet.

3. If a piece of round timber be 24 feet long, and the quarter girt be 17·75 inches, how many feet are contained therein? *Ans.* 52·51 feet.

4. How many solid feet are contained in a tree whose length is 17¼ feet, and girts in five places, *viz.* in the first 9·43 feet, in the second 7·92, in the third 6·15, in the fourth 4·74, and in the fifth 3·16 feet?

Ans. The mean girt is 6·28 feet, content 42·52 feet.

5. The dimensions of four *elm* trees, cut down in the Royal Gardens at Kensington, and sold at 2s. 7d. per foot, are as follow; required the Content and value.

No.	*Length.*		*Quarter Girt.*	*Quarter Girt allowing 1½ in. for bark.*	*Content in feet.*
	Feet.	*In.*	*Inches.*	*Inches.*	
I.	45		20	18½	……
II.	17	10	21½	20	……
	22	6	12¾	11¼	……
	13		12	10½	……
III.	10	6	33½	32	……
	17		27	25½	……
	7		22	20½	……
	14		14	12½	……
IV.	12	6	40¼	39	……
	17		33½	32	……
	13		21	19¼	……
	16		21	19½	……
	17	6	11¾	10½	……
				Total	716·087

716 feet at 2s. 7d. per Foot amounts to £92. 9s. 8d. *Answer.*

SCHOLIUM.

The 5th Problem contains the whole substance of measuring round timber as practised by all timber merchants; and though the method has been noticed as erroneous by various writers for nearly a century past, it still stands its ground; nor does it appear probable that it will soon be abolished.

In measuring *round* timber by the foregoing method of taking a quarter of the circumference in the middle, for the side of a mean square; it is objected that it makes the content *too little*, and that such timber ought to be measured as a *cylinder*, or the *false* content should be increased in the ratio of ·0625 to ·07958, or as 11 to 14.

But this objection is answered by saying, that before the wood can be squared, and made fit for use, a great part of it goes to waste in chips, and therefore the quantity of round timber ought to be reckoned no more than what the inscribed square will amount to. Now, if the circumference be 1, the area of the section will be ·07958, the square of the quarter-girt ·0625, and the side of the inscribed square ·2251, the square of which is ·05067. Therefore to make the content by the inscribed square correspond with the cylindrical content, it ought to be increased in the ratio of ·05067 to ·07958, or 7 to 11, which is a greater increase than 11 to 14, and therefore instead of the content by the quarter-girt being *too little*, it is on this consideration *too much*.

Again, when you take the mean girt of a tree, it is very probable that the tree is not perfectly circular in that place, and the more it differs from a circle the greater the quarter-girt will be; so that if you were to measure such a tree as a *cylinder*, by the proper rule for that purpose, the content would in reality be too much, on account of the error in taking the girt.

It is likewise objected, that in *round* tapering timber, taking the side of a square in the middle of the piece makes the content *too little*, and that even in a greater proportion than above. The answer given to this objection is, that in almost all cases the greater end must be cut away till it be of the same dimensions as the less end, otherwise the timber cannot be sawn into useful materials; and therefore there is no just reason for any objection to the rule on this ground.

The following rule, which was first given by Dr. *Hutton*, in the year 1770, has appeared in every treatise on mensuration that has been published since that time.

RULE. "*Multiply the square of $\frac{1}{5}$ of the girt, or*
" *circumference by twice the length, and the product*
" *will be the content (extremely near the truth).*"

When a piece of timber is *perfectly* cylindrical, this rule is *nearly* true. It is said to be full as *easy* in practice, as the quarter-girt rule; and the assertion is true with respect to finding the content of a piece of timber, but not so in taking the dimensions; for the measurer must either fold his cord into five equal parts, which cannot be done very expeditiously, or he must measure the whole length of the girt by the carpenters rule, which, when the tree is of large dimensions, will occupy more time than is usually allotted to business of this kind. It may be said that the girt may be taken with a piece of tape divided into equal parts of five inches each, and these may be subdivided into halves and quarters; but there are very few purchasers of timber who will agree to the quarter girt being measured with tape.

Another reason given for using this rule, is the preventing of the sellers from cutting trees into different lengths, so that the parts, when measured separately, shall exceed the whole; and rules have been fluxionally investigated to shew how this may be effected *.

Now

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* 1. If a piece of round tapering timber be cut through, exactly in the middle, the two parts will measure to the most possible, and their measurements, added together, will exceed the measurement of the whole, whether this rule or the quarter girt rule be used.

2. If a piece of round tapering timber can be cut where the girt is $\frac{1}{3}$ of the greatest girt, the content of the part next the greater end so cut off, will be the greatest possible. If the greatest girt be less than 3 times the least, the tree cannot be cut as required by this Problem.

3. To cut a round tapering tree, so that the part next to the

greater

Now since this rule, as well as the quarter girt rule, is inaccurate when applied to the frustum of

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greater end may measure exactly to the same quantity as the whole measures to.

To four times the sum of the girts of the two ends, add their difference, multiply this sum by the aforesaid difference, and extract the square-root of the product.

From this square-root take twice the sum of the girts diminished by their difference; then as the difference of the girts is to half the length, so is the remainder, found above, to the length which must be cut off from the smaller end.

And half the aforesaid square-root, diminished by half the sum of the girts, will give the girt at the section.

Example 1. Suppose a tree to girt 14 feet at the greater end, and 2 feet at the less, and that its length be 32 feet; the middle girt is 8 feet, and the *whole* content, by the rule in this scholium, will be 163·84 feet; and the two parts 154·88 and 32 feet, exceeding the whole by 23 feet.

2. Continuing the same example, wherein the greater girt is 14 feet, less 2 feet, and length 32 feet.

From the greater girt take 3 times the less, and divide the remainder by the difference of the girts, the quotient multiplied by $\frac{1}{3}$ of the whole length, will give the length to be cut off from the smaller end.

The distance from the less end is $7\frac{1}{9}$ feet, from the greater end $24\frac{8}{9}$, the girt at the section $4\frac{2}{3}$ feet, content of the greater end, by the rule in this scholium, is 173·4479 feet, exceeding the whole by 9·6 feet.

3. Continuing the same example, wherein the greater girt is 14 feet, less 2 feet, and length 32 feet.

The square-root mentioned in the third Note above, will be found to be 30·1993377; the length to be cut from the smaller end is 13·5991169 feet, and 18·4008831 feet from the greater end; girt at the section 7·0996688, middle girt between the section and the greater end 10·5498344 feet, and the solidity, by the rule in this scholium, is 163·84, the same as the whole.

The rules in this Note, though very curious, are of little use; for it seldom happens that a person purchases a part of a tree: and when a lot of timber is bought, the buyer generally cuts the trees into such lengths as may be most convenient for his use. The example given in this Note is a very particular one; and if such a tree were to be measured, the *seller* would divide it into four or five different lengths, so as to make each length as near a cylinder as possible. See the 5th example in this Problem.

a cone, the seller may as easily deceive the buyer when this rule is used, if he be so disposed, as when the quarter-girt rule is used.

For, if $G =$ the girt, and $L =$ the length, the content by the old rule is $\frac{G}{4} \times L$, and by this rule it is $\frac{G}{5} \times 2\ L$. Now the maximum by the one rule, will invariably be the maximum by the other; and as the only case in which deception can be used, is when the rules are applied to the frustum of a cone, where neither of them are true, it is plain that this rule will no more prevent the evil complained of than the old one.

III. OF CARPENTERS AND JOINERS WORK.

The Carpenters and Joiners works which are measurable, are flooring, partitioning, roofing, wainscotting, &c.

1. *Of Flooring.*

Joists are measured by multiplying their breadth by their depth, and that product by their length. They receive various names according to the position in which they are laid to form a floor; such as trimming joists, common joists, girders, binding joists, bridging joists, and ceiling joists.

Girders and joists of floors, designed to bear great weights, should be let into the walls at each end about $\frac{2}{3}$ of the wall's thickness.

In boarded flooring, the dimensions must be taken to the extreme parts, and the number of squares of 100 feet must be calculated from these dimensions. Deductions must be made for stair-cases, chimneys, &c.

Example 1. If a floor be 57 feet 3 inches long, and 28 feet 6 inches broad; how many squares of flooring are there in that room?

By

By Decimals.	*By Duodecimals.*
57·25	F. I.
28·5	57 : 3
	28 : 6
28625	
45800	456
11450	114
	28 : 7 : 6
100)1631·625 feet.	7 : 0 : 0
Squares 16·31625	16|31 : 7 : 6

Ans. 16 squares and 31 feet.

2. Let a floor be 53 feet 6 inches long, and 47 feet 9 inches broad; how many squares are contained in that floor? *Ans.* 25 squares 54 feet.

3. A floor being 36 feet 3 inches long, and 16 feet 6 inches broad; how many squares are contained in it? *Ans.* 5 squares 98 feet.

4. In a naked floor the girder is 1 foot 2 inches deep, 1 foot broad, and 20 feet long; there are 8 bridging joints, whose *scantlings* (viz. breadths and depths) are 3 inches by 6½ inches, and 20 feet long; 8 binding joists, whose lengths are 9 feet, and scantlings 8½ inches by four inches: the ceiling joists are 24 in number, each 6 feet long, and their scantlings 4 inches by 2½ inches. Required the solidity of the whole? *Ans.* 72 feet.

5. Suppose a house of three stories, besides the ground floor, was to be floored at £6. 10s. per square; the house measures 20 feet 8 inches, by 16 feet nine inches, there are 7 fire-places, the measures whereof are two, each of 6 feet by 4 feet 6 inches; two others, each of 6 feet by 5 feet 4 inches; and two, each of 5 feet 8 inches by 4 feet 8 inches; and the seventh, 5 feet 2 inches by 4 feet: the well-hole for the stairs is 10 feet 6 inches by 8 feet 9: what will the whole come to?

Ans. £53. 13s. 3¼d.

2. *Of*

2. *Of Partitioning.*

Boarded partitions are measured by the square in the same manner as flooring, and deductions must be made for doors and windows, except they are included by agreement.

The strongest partitions are those made with framed timber, all the parts of which are measured as in flooring, except the king-posts, and these are of the same kind as in roofing, an example of which will be given in the next article.

Example 1. If a partition between two rooms be in length 82 feet 6 inches, and in height 12 feet 3 inches, how many squares are contained therein?

Ans. 10 squares 10 feet.

2. If a partition between two rooms be in length 91 feet 9 inches, and its breadth 11 feet 3 inches; how many squares are contained in it?

Ans. 10 squares 32 feet.

3. *Of Roofing.*

In roofing take the whole length of the timber for the length of the framing, and for the breadth gird over the ridge from wall to wall with a string. This length and breadth must be multiplied together for the content.

It is a rule also among workmen, that the flat of any house, and half the flat thereof, taken within the walls, is equal to the measure of the roof of the same house; but this is when the roof is of a true pitch. The pitch of every roof ought to be made according to its covering, which in *England* is lead, pantiles, plain-tiles, or slates. The usual pitches, are the pediment pitch, used when the covering is lead; the perpendicular height is $\frac{2}{9}$ of the breadth of the building. The common, or true pitch, where the length of the rafters are $\frac{3}{4}$ of the breadth of the building; this is used when the covering is plain-tiles. The gothic pitch, is when the length of the principal rafters is equal to the breadth of the building, forming an equilateral triangle: this pitch is used when the covering is of pantiles.

In the measuring of roofing for workmanship alone, all holes for chimney-shafts and sky-lights are generally deducted. But in measuring for work and materials, they commonly measure in all sky-lights, luthern-lights, and holes for the chimney-shafts, on account of their trouble and waste of materials.

Example 1. If a house within the walls be 44 feet 6 inches long, and 18 feet 3 inches broad, how many squares of roofing will cover that house?

By Decimals.	*By Duodecimals.*
18·25	F. I.
44·5	44 : 6
	18 : 3
9125	352
7300	44
7300	11 : 1 : 6
Flat 812·125	9 : 0 : 0
Half 406·	The flat 812 : 1 : 6
100)1218·	The half 406
12·18	Sum 12|18

Ans. 12 squares 18 feet.

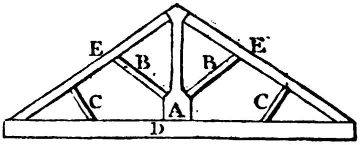
2. What cost the roofing of a house at 10s. 6d. per square; the length, within the walls, being 52 feet 8 inches, and the breadth 30 feet 6 inches; the roof being of a true pitch? *Ans.* £12. 12s. 11¼d.

3. The roof of a house is of a true pitch: and the house measures 40 feet 6 inches in length,

within the walls, and 20 feet 6 inches in breadth; how many squares of roofing are contained therein?

Ans. 12·45375.

Note. All timbers in a roof are measured in the same manner as in floors, except king-posts, &c. such as A in the annexed figure, where there is a necessity for cutting out parallel pieces of wood from the sides, in order that the ends of the braces B, that come against them, may have what the workmen call a square butment. *To measure king-posts for workmanship,* take their breadth and depth at the widest place, and multiply these together, and the product by the length. *To find the quantity of materials,* if the pieces sawn out are 2½ inches broad, or upwards, and more than 2 feet long, they are considered as pieces of timber fit for use. In measuring these pieces, the shortest length must always be taken, because the sawing of them from the king-post, renders a part of them useless. The solidity of these pieces must be deducted from the solidity of the king-post. When the pieces sawn out are of smaller dimensions than above described, the whole post is measured as solid without any deduction, the pieces cut out being esteemed as of little or no value.



4. Let the tie-beam D, in the above figure, be 36 feet long, 9 inches broad, and 1 foot 2 inches deep; the king-post A 11 feet 6 inches high, 1 foot broad at the bottom, and 5 inches thick; out of this post are sawn two equal pieces from the sides, each 7 feet long and 3 inches broad. The braces B, B, are 7 feet 6 inches long, and 5 inches by 5 inches square: the rafters E, E, are 19 feet long, 5 inches broad, and 10 inches deep: the struts C, C, are 3

feet 6 inches long, 4 inches broad, and 5 inches deep. Required the measurement for workmanship, and likewise for materials?

Answer.

F.	I.	P.	
31 :	6 :	0	solidity of the tie-beam D.
4 :	9 :	6	solidity of the king-post A.
2 :	7 :	3	solidity of the braces B, B.
13 :	2 :	4	solidity of the rafters E, E.
— :	11 :	8	solidity of the struts C, C.
53 :	0 :	9	solidity for workmanship.
1 :	5 :	6	solidity cut from the king-post.
51 :	7 :	3	solidity for materials.

4. *Of Wainscotting, &c.*

Wainscotting is measured by the yard square, consisting of 9 square feet. The dimensions are taken in feet and inches; thus in taking the height of a room they girt over the cornice, swelling pannels, and mouldings, with a string, and for the compass of the room, they measure round the floor. Doors, window shutters, and such like, where both sides are planed, are considered as work and half; therefore in measuring a room, they need not be deducted, but the room may be measured as if there were none; then the contents of the doors and shutters must be found, and the half thereof added to the content of the whole room.

Windows, where there are no shutters, must be deducted; also chimnies, window-seats, cheek-boards, sopheta-boards, linings, &c. must be measured by themselves.

Windows are generally made and valued by the foot superficial measure, and sometimes at so much per window. When they are measured, the dimensions must be taken in feet and inches, from the under side of the sill to the upper side of the top-rail, for the height; and for the breadth, from outside to outside of the jambs. This length, multiplied by the breadth, will be the superficies.

Weather-boarding is measured by the yard square, and sometimes by the square.

Stair-cases are measured by the foot superficial, and the dimensions are taken with a string, girt over the *riser* and *tread*, and that length, or girt, multiplied by the length of the step, will give the superficies.

The rail is taken at so much per foot in length, according to the diameter of the well-hole; architrave string-boards by the foot superficial; brackets and strings at so much per piece, according to the workmanship.

Door-cases are measured by the foot superficial, and the dimensions must be taken with a string, girt round the architrave and inside of the jambs, for the breadth; and for the length, add the length of the two jambs to the length of the cap piece, (taking the breadth of the opening for the length) the product of these two will be the superficies.

Frame-doors are measured by the foot, or sometimes by the yard square.

Modillian cornices, coves, &c. are generally measured by the foot superficial.

Frontispieces are measured and valued by the foot superficial, and every part is measured separately, viz. architrave, frieze, and cornice.

The following articles are generally paid for by lineal measure, viz. beads, stops, artragals, cappings, fillets, boxings to windows, skirting boards, and water trunks.

Example 1. If a room, or wainscot, being girt downwards over the mouldings, be 15 feet 9 inches high, and 126 feet 3 inches in compass; how many yards does that room contain?

By

By Duodecimals.	*By Decimals.*
F. I.	126·25
126 : 3	15·75
15 : 9	
	63125
630	88375
126	63125
63 : 1 : 6	12625
31 : 6 : 9	
3 : 9 : 0	9)1988·4375
9)1988 : 5 : 3	220·8
Ans. 220·8.	*Ans.* 220 yards 8 feet.

2. If a room of wainscot be 16 feet 3 inches high, and the compass of the room 137 feet 6 inches; how many yards are contained in it?

Ans. 248 yards 2 feet.

3. If the window shutters about a room be 69 feet 9 inches broad, and 6 feet 3 inches high; how many yards are contained therein at work and half?

Ans. 72·656 yards.

4. What will the wainscotting of a room come to at 6 shillings per square yard, supposing the height of the room, including the cornice and moulding, be 12 feet 6 inches, and the compass 83 feet 8 inches; three window-shutters, each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches; the shutters and doors being worked on both sides, are reckoned work and half?

Ans. £36. 12s. 2½d.

5. A rectangular room measures 129 feet 6 inches round, and is to be wainscotted at 3s. 6d. per square yard: after due allowance for girt of cornice, &c. it is 16 feet 3 inches high; the door is 7 feet by 3 feet 9 inches; the window-shutters, two pair, are 7 feet 3 inches by 4 feet 6 inches; the cheek-boards round them come 15 inches below the shutters, and are 14 inches in breadth; the lining-boards round

the door-way are 16 inches broad; the door and window-shutters being worked on both sides, are reckoned as work and half, and paid for accordingly; the chimney 3 feet 9 inches by 3 feet, not being enclosed, is to be deducted from the superficial content of the room. The estimate of the charge is required. *Ans.* £43. 4s. $6\frac{1}{4}$d.

IV. OF BRICKLAYERS WORK.

The principal is tiling, slating, walling, and chimney-work.

1. *Of Tiling, or Slating.*

Tiling and slating are measured by the square of 100 feet, as flooring, partitioning, and roofing were in the Carpenters work; so that there is not much difference between the roofing and tiling; yet the tiling will be the most; for the bricklayers sometimes will require to have double measure for hips and valleys.

When gutters are allowed double measure, the way is to measure the length along the ridge-tile, and add it to the content of the roof: this makes an allowance of one foot in breadth, the whole length of the hips or valleys. It is usual also to allow double measure at the eaves, so much as the projector is over the plate, which is commonly about 18 or 20 inches.

Sky-lights and chimney-shafts are generally deducted, if they be large, otherwise not.

Example 1. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 37 feet 3 inches, and the length 45 feet; how many squares of tiling are contained therein?

By Duodecimals.	*By Decimals.*
F. I.	37·25
37 : 3	45
45 : 0	
	18625
185	14900
148	
11 : 3	16|76·25
16|76 : 3	

Ans. 16 squares, 76 feet.

2. There is a roof covered with tiles, whose depth on both sides, (with the allowance at the eaves) is 35 feet 9 inches, and the length 43 feet 6 inches; how many squares of tiling are in the roof?
Ans. 15 squares 55 feet.

3. What will the tiling of a barn cost at £1. 5s. 6d. per square, the length being 43 feet 10 inches, and breadth 27 feet 5 inches, on the flat, the eaves projecting 16 inches on each side? *Ans.* £24. 9s. 5½d.

2. *Of Walling.*

Bricklayers commonly measure their work by the rod of 16½ feet, or 272¼ square feet. In some places it is a custom to allow 18 feet to the rod; that is, 324 square feet. Sometimes the work is measured by the rod of 21 feet long and 3 feet high, that is, 63 square feet; and then no regard is paid to the thickness of the wall in measuring, but the price is regulated according to the thickness.

When you measure a piece of brick-work, the first thing is to enquire by which of these ways it must be measured; then having multiplied the length and breadth in feet together, divide the product by the proper divisor, viz. 272·25; 324; or 63; according to the measure of the rod, and the quotient will be the answer in square rods of that measure.

But commonly brick walls that are measured by the rod, are to be reduced to a standard thickness of a brick and a half, which may be done by the following

RULE.

Multiply the number of superficial feet that are contained in the wall by the number of half bricks which that wall is in thickness; one third part of that product will be the content in feet.

The dimensions of a building are generally taken by measuring half round the outside and half round the inside, for the whole length of the wall; this length, being multiplied by the height, gives the superficies. And to reduce it to the standard thickness, &c. proceed as above. All the vacuities, such as doors, windows, window-backs, &c. must be deducted.

To measure any arched way, arched-window, or door, &c. take the height of the window, or door, from the crown or middle of the arch, to the bottom or sill; and likewise from the bottom or sill to the spring of the arch, that is, where the arch begins to turn. Then to the latter height, add twice the former, and multiply the sum by the width of the window door, &c. and $\frac{1}{3}$ of the product will be the area, sufficiently near for practice.

Example 1. If a wall be 72 feet 6 inches long, and 19 feet 3 inches high, and 5 bricks and a half thick; how many rods of brick-work are contained therein, when reduced to the standard?

By Decimals.

19·25
72·5

9625
3850
13475

1395·625
11

3)15351·875

272·25)5117·291(18 rods

239479

68·06)216·79(3 quarters

12·61 feet.

By Duodecimals.

F. I.
72 : 6
19 : 3

648
72
18 : 1 : 6
9 : 6 : –

1395 : 7 : 6
11

3)15351 : 10 : 6

272)5117 : (18 rods.

2397

68)221(3 quarters.

17 feet.

Note. That 68·06 is one-fourth part of 272·25, and 68 is one-fourth of 272.

In reducing feet into rods, it is usual to divide by 272, *rejecting the decimal* ·25; by this method, the answer is 18 rods 3 quarters and 17 feet; about 4½ feet too much.

2. If a wall be 245 feet 9 inches long, 16 feet 6 inches high, and two bricks and a half thick; how many rods of brick-work are contained therein, when reduced to the standard thickness?

Ans. 24 rods 3 quarters 24 feet.

3. How many rods are contained in a wall 63½ feet long, 14 feet 11 inches high, and 2½ bricks in thickness, when reduced to the standard?

Ans. 5 rods 218 feet.

4. A triangular gable-end is raised to the height of 15 feet above the end wall of a house, whose width is 45 feet, and the thickness of the wall is 2½ bricks; required the content in rods at standard thickness? *Ans.* 2 rods 18 feet.

5. Admit the end wall of a house to be 28 feet 10 inches in breadth, and the height of the roof from the ground 55 feet 8 inches, the gable (or triangular part above the side-walls) to rise 42 courses of bricks, reckoning 4 courses to a foot; and that 20 feet high be 2½ bricks thick, 20 feet more 2 bricks thick, and the remaining 15 feet 8 inches, 1½ brick thick; what will the work come to at £5. 16s. per rod, the gable being one brick in thickness?

Ans. £48. 13s. 5¼d.

3. *Of Chimnies.*

If a chimney stand by itself, without any party-wall being adjoined, take the girt in the middle for the length, and the height of the story for the breadth; the thickness must be the same as the depth of the jambs, if the chimney be wrought upright from the mantle-tree to the ceiling, and no deduction must be made for the vacancy between the floor (or hearth) and the mantle-tree, because of the gatherings of the breast and wings, to make room for the hearth in the next story.

If the chimney-back be a party-wall, and the wall be measured by itself, then you must measure the depth of the two jambs, and the length of the breast for a length, and the height of the story for the breadth; the thickness will be the same as the depth of the jambs.

Chimney-shafts, viz. that part which appears above the roof, are measured by girding them round the middle for the length, and taking the height for the breadth.

The thickness is generally reckoned half a brick more than it really is, in consideration of plastering, and the trouble of scaffolding. In some places double measure is allowed for workmanship, on account of extra trouble.

Example 1. Suppose the dimensions of a chimney having a double funnel towards the top, and a double shaft, be as follow, viz. in the parlour, the breast and the two jambs measure 18 feet 9 inches, and the height of the room is 12 feet 6 inches; in the first floor, the breast and the two jambs girt 14 feet 6 inches, and the height is 9 feet; in the second floor, the breast and the jambs girt 10 feet 3 inches, and the height is 7 feet; above the roof, the compass of the shaft is 13 feet 9 inches, and its height 6 feet 6 inches; lastly, the length of the middle partition, which parts the funnels, is 12 feet, and its thickness 1 foot 3 inches: how many rods of brickwork, standard measure, are contained in the chimney, allowing double measure?

F. I.
1. 18 : 9
12 : 6

225 : 0
9 : 4 : 6

234 : 4 : 6

F. I.
2. 14 : 6
9 : –

130 : 6

F. I.
3. 10 : 3
7 : –

71 : 9

F. I.
4. 13 : 9
6 : 6

82 : 6
6 : 10 : 6

89 : 4 : 6

F. I.
1 : 3
12 : –

15 : 0 partition.

F. I. Pts.
234 : 4 : 6 parlour.
130 : 6 : – first floor.
71 : 9 : – sec. floor.
89 : 4 : 6 shaft.
15 : – : – partition.

Feet 541 : – : – sum.

Feet 1082 : – : – double.

272)1082(3 rods.

68)266(3 quarters.

62 feet.

Ans. 3 rods, 3 quarters, and 62 feet, admitting the common thickness to be 1½ brick.

Note. This chimney being measured as if it were solid, no vacuity, or opening, for the fire-place, is allowed for, in any of the floors.

2. Suppose the breadth of the breast of a chimney, not standing in an angle, to be 7 feet 3 inches, the depth of the jambs 3 bricks thick; the height of the room 14 feet 6 inches; the vacuity for the fire-place 4 feet high, 3 feet 6 inches wide, and 3 bricks deep. The chimney and fire-place in each of the other two rooms are of the same dimensions, but the height of the first floor is 12 feet; and the jamb, which is the same height as the story, 10 feet 6 inches. The shaft stands 6 feet above the roof, and its compass is 10 feet; also its thickness is estimated at $1\frac{1}{2}$ brick: how many rods of brick-work standard thickness, allowing double measure, are contained in the chimney, deducting the three fire-places? *Ans.* 3 rods, 3 quarters, and 5 feet.

V. OF PLASTERERS WORK.

Plasterers work is principally of two kinds; namely, plastering upon laths, called *ceiling*, and plastering upon walls or partitions made of framed timber, called *rendering*.

In plastering upon walls, no deductions are made except for doors and windows, because *cornices*, *festoons*, enriched mouldings, &c. are put on after the room is plastered.

In plastering timber partitions, in large warehouses, &c. where several of the braces and larger timbers project from the plastering, a fifth part is commonly deducted. Plastering between these timbers is generally called rendering between quarters.

Whitening and colouring are measured in the same manner as plastering; and in timbered partitions, one-fourth, or one-fifth of the whole area is commonly added, for the trouble of colouring the sides of the quarters and braces.

Plasterers work is measured by the yard square, consisting of 9 square feet. In arches, the girt round them, multiplied by the length, will give the superficies.

Example 1. If a ceiling be 59 feet 9 inches long, and 24 feet 6 inches broad; how many yards does that ceiling contain?

By Duodecimals.	*By Decimals.*
F. : I.	59·75
59 : 9	24·5
24 : 6	
236	29875
118	23900
29 : 10 : 6	11950
18 : 0 : 0	9)1463·875 feet.
9)1463 : 10 : 6	*Answer* 162·65 yards.
162 yards 5 feet.	

2. If the partitions between rooms be 141 feet 6 inches round, and 11 feet 3 inches high; how many yards are in those partitions? *Ans.* 176·87 inches.

3. What will be the expense of plastering a ceiling, at 11½d. per yard, supposing the length 22 feet 7 inches, and breadth 13 feet 11 inches?

Ans. £1. 13s. 5½d.

4. There is a partition which measures 234 feet 8 inches round, and 14 feet 6 inches high; this partition is rendered between quarters, that is, it is made of framed timber, and the interstices are filled up with lath and plastering. The lathing and plastering will be 8d. per yard, and the whitening 2d. per yard, what will the whole come to?

Ans. £13. 17s. 2¼d.

5. The length of a room is 14 feet 5 inches, breadth 13 feet 2 inches, and height 9 feet 3 inches to the under side of the cornice, which projects 5 inches

from the wall, on the upper part next the ceiling; required the quantity of rendering and plastering, there being no deductions but for one door, the size whereof is 7 feet by 4?

Ans. { 53 yards 5 feet of rendering, and
18 yards 5 feet of ceiling.

6. The circular vaulted roof of a church measures 105 feet 6 inches in the arch, and 275 feet 5 inches in length; what will the plastering come to at 1s. per yard. *Ans.* £150. 17s. 5¼d.

VI. OF PAINTERS WORK.

Painters generally calculate the contents of large articles; such as wainscotting, stuccoed walls, doors, and window-shutters, by the square yard; and every part upon which the colour is laid is measured.

Cornices and mouldings are estimated by the lineal foot, window-frames and reveals at a fixed price for each, sash-frames at a certain price per dozen, according to their size; sky-lights, window-bars, casements, &c. are charged per piece. Ballustrades are generally measured by taking the length of the hand-rail for one dimension; and twice the height of the baluster upon the landing, added to the girt of the hand-rail, for the other dimension.

For trellis work, double the area of one side, is sometimes taken for the measure of both sides; but no general rule can be given for measuring them.

Painters proportion their prices to the nature of the colouring, and to the number of coats which the work receives; and in measuring wainscotting, stuccoed walls, doors and window shutters, they always gird over the projecting pannels, and press the tape into all the mouldings. Deductions are always made for fire-places and other openings.

Example 1. If a room be painted, whose height (being girt over the mouldings) is 16 feet 6 inches, and the compass of the room 97 feet 9 inches; how many yards are in that room?

By Duodecimals.	*By Decimals.*
F. I.	
97 : 9	97·5
16 : 6	16·5
584	48875
98	58650
48 : 10 : 6	9775
9)1612 : 10 : 6	9)1612·875
179 : 1	179·2

Answer 179 yards 2 feet, nearly.

2. A gentleman had a room painted at 8½d. per yard, the measure whereof is as follows: the height 11 feet 7 inches, the compass 74 feet 10 inches, the door 7 feet 6 inches by 3 feet 9 inches; five window-shutters, each 6 feet 8 inches by 3 feet 4 inches, the breaks in the windows 14 inches deep and 8 feet high; the opening for the chimney 6 feet 9 inches by 5 feet, to be deducted; the shutters and doors are coloured on both sides; what will the whole come to? *Ans.* £4. 6s. 11d.

3. Suppose a room were to be painted, and that its length is 24 feet 6 inches, breadth 16 feet 3 inches, and height 12 feet 9 inches; also the size of the door 7 feet by 3 feet 6 inches, and the size of the window-shutters to each of the windows, there being two, is 7 feet 9 inches by 3 feet 6 inches; but the breaks of the windows themselves are 8 feet 6 inches high, and 1 foot 3 inches deep; what will be the expense of giving it three coats, at 2d. per yard each; the size of the fire-place to be deducted, being 5 feet by 5 feet 6 inches? *Ans.* £3. 3s. 10½d.

VII. OF GLAZIERS WORK.

Glaziers take their dimensions in feet, inches, and eighths or tenths, or else in feet, and hundredth parts of a foot, and estimate their work by the square foot.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying its superficies by the number of panes. But more generally, they measure the length and breadth of the window over all the panes and their frames, for the length and breadth of the glazing.

Circular or oval windows, as fan-lights, &c. are measured as if they were square, taking for their dimensions the greatest length and breadth; as a compensation for the waste of glass, and labour in cutting it to the necessary forms.

Example 1. If a pane of glass be 4 feet 8 inches and 3 quarters long, and 1 foot 4 inches and 1 quarter broad; how many feet of glass are in that pane?

By Duodecimals.	*By Decimals.*
F. I. P.	4·729
4 : 8 : 9	1·354
1 : 4 : 3	
	18916
4 : 8 : 9	23645
1 : 6 : 11 : 0	14187
1 : 2 : 2 : 3	4729
6 : 4 : 10 : 2 : 3	6·403066

Answer, 6 feet 4 inches.

2. If there be 8 panes of glass, each 4 feet 7 inches 3 quarters long, and 1 foot 5 inches 1 quarter broad; how many feet of glass are contained in the said 8 panes? *Ans.* 53 feet 5 inches.

3. If there be 16 panes of glass, each 4 feet 5 inches and a half long, and 1 foot 4 inches 3 quar-

ters broad; how many feet of glass are contained in them? *Ans.* 99 feet 6 inches.

4. If a window be 7 feet 3 inches high, and 3 feet 5 inches broad; how many square feet of glazing are contained therein? *Ans.* 24 feet 9 inches.

5. There is a house with 3 tiers of windows, 7 in a tier; the height of the first tier is 6 feet 11 inches, of the second 5 feet 4 inches, and of the third 4 feet 3 inches; the breadth of each window is 3 feet 6 inches; what will the glazing come to at 14½d. per foot? £24. 8s. 5¼d.

6. What will the glazing a triangular sky-light come to at 10d. per foot; the base being 12 feet 6 inches long, and the perpendicular height 16 feet 9 inches? *Ans.* £4. 7s. 2¼d.

7. What is the area of an elliptical fan light, of 14 feet 6 inches in length, and 4 feet 9 inches in breadth? *Ans.* 68 feet 10 inches.

VIII. OF PLUMBERS WORK.

Plumbers work is generally rated at so much per pound, or by the hundred weight of 112 pound, and the price is regulated according to the value of lead at the time when the work is performed.

Sheet lead, used in roofing, guttering, &c. weighs from 6 to 12 pound per square foot, according to the thickness; and leaden pipe varies in weight per yard, according to the diameter of its bore in inches.

The following table shews the weight of a square foot of sheet lead, according to its thickness, reckoned in parts of an inch; and the common weight of a yard of leaden pipe corresponding to the diameter of its bore in inches.

Thickness of Lead.	*Pounds to a Square Foot.*	*Bore of Leaden Pipe.*	*Pounds per Yard.*
$\frac{1}{10}$	5·899	$\frac{3}{4}$	10
$\frac{1}{9}$	6·554	1	12
$\frac{1}{8}$	7·373	$1\frac{1}{4}$	16
$\frac{1}{7}$	8·427	$1\frac{1}{2}$	18
$\frac{1}{6}$	9·831	$1\frac{3}{4}$	21
$\frac{1}{5}$	11·797	2	24

Example 1. A piece of sheet lead measures 16 feet 9 inches in length, and 6 feet 6 inches in breadth; what is its weight at 8¼ pound to a square foot?

By Duodecimals.	*By Decimals.*
F. I.	*Feet.*
16 : 9	16·75
6 : 6	6·5
100 : 6	8375
8 : 4 : 6	10050
108 : 10 : 6	108·875 *feet.*

Then 1 foot : 8¼ pounds :: 108·875 feet : 898·21875 pounds = 8 cwt. 2¼ pounds, nearly.

2. What weight of lead $\frac{1}{10}$ of an inch thick will cover a flat, 18 feet 6 inches long, and 12 feet 3 inches broad, the lead weighing 6 pounds to the square foot? *Ans.* 12 cwt. 15¾ pounds.

3. What will be the expense of 120 yards of leaden pipe of an inch and a half bore, at 3d. per pound, admitting each yard to weigh 18 pounds.
Ans. £27.

4. What cost the covering and guttering a roof with lead, at 18s. per hundred weight; the length

of the roof being 43 feet, and the girt over it 32 feet; the guttering being 57 feet in length and 2 feet in breadth; admitting a square foot of lead to weigh 8¼ pounds? *Ans.* £104. 15s. 3¼d.

IX. OF MASONS WORK.

Masons measure their work sometimes by the foot solid, sometimes by the foot superficial, and sometimes by the foot in length. In taking dimensions they girt all their mouldings as joiners do.

The solids consist of blocks of marble, stone, pillars, columns, &c. The superficies are pavements, slabs, chimney-pieces, &c.

Masons reckon all such stones as are above 2 inches thick, at so much per foot, solid measure. And, for the workmanship, they measure the superficies of that part of the stone which appears without the wall.

Example 1. If a wall be 97 feet 5 inches long, 18 feet 3 inches high, and 2 feet 3 inches thick; how many solid feet are contained in that wall?

By Duodecimals.	*By Decimals.*
F. : I.	
97 : 5 length	97·417
18 : 3 breadth or height.	18·25
776	487085
97	194834
24 : 4 : 3	779336
6 : 0 : 0	97417
1 : 6 : 0	
	1777·86025
1777 : 10 : 3 superficies.	2·25
2 : 3 thickness.	
	888930125
3555 : 8 : 6	355572050
444 : 5 : 6 : 9	355572050
4000 : 2 : 0 : 9 solidity.	4000·1855625

2. If a wall be 107 feet 9 inches long, and 20 feet 6 inches high; how many superficial feet are contained therein? *Ans.* 2208 feet 10 inches.

3. If a wall be 112 feet 3 inches long, and 16 feet 6 inches high; how many superficial rods of 63 square feet each are contained therein? *Ans.* 29 rods 25 feet.

4. What is a marble slab worth, whose length is 5 feet 7 inches, and breadth 1 foot 10 inches, at 6s. per foot superficial? *Ans.* £3. 1s. 5d.

X. OF PAVERS WORK.

Pavers work is measured by the square yard, consisting of 9 square feet. The superficies is found by multiplying the length by the breadth.

Example 1. What cost the paving a foot-path at 3s. 4d. per yard; the length being 35 feet 4 inches, and breadth 8 feet 3 inches?

By Decimals.	*By Duodecimals.*
	F. : I.
35·3333 &c.	35 : 4
8¼	8 : 3
282.6666	282 : 8
8·8333	8 : 10
9)291·4999 &c.	9)291 : 6
3s. 4d. is ⅙)32·3888 &c.	3s. 4d. is ⅙)32 | 3 : 6
£5·3981	Feet - - - 5 6 8
20	3 = ⅓ of a yard 0 1 1¼·33
s. 7·9620	6 = ⅙ of 3 feet 0 0 2·88
12	£5 7 11½·21
d. 11·5440	
4	
2·1760	

Ans. £5. 7s. 11½d.

2. What will the paving a court-yard come to at 3s. 4d. per yard, the length being 24 feet 5 inches, and breadth 12 feet 7 inches? *Ans.* £.5 13s. 9½d.

3. What will be the expense of paving a rectangular court yard, its length being 62 feet 7 inches, and breadth 44 feet 5 inches; and in which there is laid a foot-path the whole length of it, 5½ feet broad, with flat stones at 3s. per yard, the rest being paved with pebbles at 2s. 6d. per yard?
Ans. £.39 11s. 3¼d.

XI. OF VAULTED AND ARCHED ROOFS, &c.

ARCHED Roofs are either vaults, domes, saloons, or groins.

Vaulted roofs are formed by arches springing from two parallel walls and meeting in a line over the middle of the building, such are the middle aisles of several churches.

Domes are made by arches springing from a circular, or polygonal base, and meeting in a point directly over the center of that base.

Saloons are formed by arches connecting the side walls of a building to a flat roof, or ceiling.

Groins are formed by the intersection of vaulted roofs with each other.

They are of various kinds, but the most common are formed by two hollow semicylinders, or two hollow semielliptical prisms crossing each other at right angles, in either of which cases, the arches spring over a square base. Groins are generally measured like a parallelopipedon, and the content is found by multiplying the length and breadth of the base by the height. Some measures deduct one-tenth of the solidity thus found, from the whole, and reckon the remainder as the solidity of the vacuity.

Vaulted Roofs are either circular, elliptical, or Gothic.

Circular Roofs are those of which the arch is a part of the circumference of a circle.

Elliptical Roofs are those of which the arch is a part of the circumference of an ellipsis.

Gothic Roofs are formed by the meeting of two equal circular arches, exactly above the span of the arch.

PROBLEM I.

To find the solidity of a circular, elliptical, or gothic, Vaulted Roof.

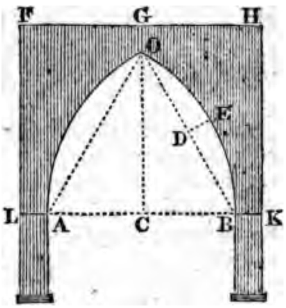
RULE. Multiply the area of one end, by the length of the roof, or vault, and the product will be the content.

To find the solidity for the materials. From the solid content of the whole arch, take the solid content of the vacuity, and the remainder will be the solidity of the arch.

Note. If the arch be a segment of a circle, the area of the end may be found by Problem XVI. or XVII. page 55 or 59.—If the arch be the segment of an ellipsis, multiply the span by the height, and that product by ·7854 for the area of the end. If it be a gothic arch, find the area of an isosceles triangle whose base is equal to the span of the arch, and its sides equal to the two chords of the circular segment of the arch; the areas of the two segments, added to the area of the triangle, will give the area of the end.

Example 1. What is the solid content of the vacuity A O E B of a Gothic vault, whose span A B is 50 feet, the chord A O, or B O, of each arch 50 feet; the distance of each arch from the middle of the chords as D E = 10 feet, and the length of the vault 20 feet?

The



The triangle A O B in this example is equilateral, and its area, by the note page 32, $= \frac{1}{4}\,\text{AB}^2\sqrt{3} = 625\sqrt{3} = 1082{\cdot}53175$.

By rule 4 page 56, note. $\frac{2}{3}(\text{BO} \times \text{DE}) + \dfrac{\text{DE}^3}{2\,\text{BO}} = \frac{2}{3}(50 \times 10) + \dfrac{10^3}{50 \times 2} = 343\frac{1}{3}$ area of the segment O E B.

Then $(1082{\cdot}53175 + 686\frac{2}{3}) \times 20 = 35383{\cdot}968$ the solidity required.

2. Let F H K L represent the perpendicular section of a Gothic Roof. The span A B = 50 feet, the thickness of the wall L A or B K at the spring of the arch = 3 feet; the thickness O G at the crown of the arch = 2 feet 4 inches, the length of the roof = 120 feet, the chord A O = B O = 50 feet, and the versed sine D E = 10 feet: required the solidity of the materials of the arch.

First $\sqrt{\text{OB}^2 - \text{BC}^2} = \sqrt{50^2 - 25^2} = 5\sqrt{75} = 43{\cdot}30127 = \text{CO}$ the height of the vacuity of the arch, and $\text{CO} + \text{OG} = 43{\cdot}30127 + 2{\cdot}33333 = 45{\cdot}6346 = \text{CG} = \text{LF}$, also $\text{AB} + \text{LA} + \text{BK} = 50 + 3 + 3 = 56 = \text{LK}$ or FH. Then $\text{LF} \times \text{FH} = 45{\cdot}6346 \times 56 = 2555{\cdot}5376$ feet the area of the section F H K L, and the area of the vacuity, by the preceding example $= 1769{\cdot}1984$ feet, hence $(2555{\cdot}5376 - 1769{\cdot}1984) \times 120 = 94360{\cdot}704$ feet, the solidity of the materials.

3. What is the solid content of the vacuity of an elliptical vault, whose span is 30 feet, and height 15 feet, the length of the vault being 90 feet?

Ans. 31808·7 feet.

4. What is the solidity of the vacuity of the middle arch of Westminster Bridge at low water? The arch is a semicircle which springs 2 feet above low water, its span is 76 feet, and length 44 feet.

Ans. 106490·3488 feet.

5. Required the solidity of the vacuity of a circular vault, and also the solidity of the materials of the vault; the span is 18 feet, the thickness of the walls at the spring of the arch 3 feet, the height of the arch 9 feet, the thickness at the crown 2 feet, and the length of the vault 50 feet.

Ans. $\begin{cases} \text{6361·74 feet solidity of the vacuity.} \\ \text{6838·26 feet solidity of the materials.} \end{cases}$

6. Required the solidity of the vacuity of a Gothic Roof, and also the solidity of the materials of the roof; the span A B is 30 feet, the chords of each arch A O and B O = 40 feet, the versed sine D E = 6 feet, the thickness L A or B K of the pier 17 feet at the spring of the arch, the thickness O G of the arch 4 feet, and the length of the roof 60 feet.

Ans. $\begin{cases} \text{52896·89325 feet solidity of the vacuity.} \\ \text{104854·11795 feet solidity of the materials.} \end{cases}$

PROBLEM II.

To find the concave, or convex surface, of circular, elliptical, or gothic Vaulted Roofs.

Rule. Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

Note. If the convex surface of the vault be required, stretch a small cord over the convexity of the vault, this cord measured will give the length of the curve. For the concave surface, measure the span and height of the arch, from which find the length of the arch by Problem XII. page 46, or Problem XXIV. page 74.

Example 1. The span of the arch of a circular vault is 50·8 feet, the height 17·0646 feet, and the length of the vault 50 feet, required the surface.

Ans. { The length of the arch is $64\frac{2}{3}$ feet, see example 5, page 48.
And the surface $3233\frac{1}{3}$ feet.

2. What is the concave surface of the bridge over the River Taafe, near Llantrissent, which is a segment of a circle, the span is 140 feet, the height 35 feet, and the width of the bridge 11 feet?

Ans. 1782·363 feet.

PROBLEM III.

To find the solid content of a dome, having the height and the dimensions of its base given.

RULE. Multiply the area of the base by the height, and $\frac{2}{3}$ of the product will be the solidity [*].

Example 1. What is the solidity of a dome, in the form of a hemisphere, the diameter of the circular base being 60 feet?

$60^2 \times {\cdot}7854 = 2827{\cdot}44$ area of the base.

Then $\frac{2}{3}$ $(2827{\cdot}44 \times 30) = 56548{\cdot}8$ cubic feet, *Ans.*

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[*] This rule is given in *Robertson's* mensuration, and has been copied into every treatise of mensuration that has been published during the present, and nearly half of the last century, without any attempt to shew how far it is correct; even those who have professed to demonstrate all their rules have, with singular indifference, neglected this.

It is plain that the rule is true only in one case, and that is when the dome is circular and its height equal to the radius of its base; for the solidity of a sphere is $\frac{2}{3}$ of that of a cylinder having the same base and height (*Keith's Geom.* 21 of X.), and the solidity of a cylinder is found by multiplying the area of the base by the height. Domes are of various shapes, as circular, elliptical, polygonal, &c. and also of various heights and their sides of different curvature; so that no general rule can be given to answer every case. When the curved sides of the dome, from the circumference of the base to the centre at the top, are circular or elliptical quadrants, and the height of the dome equal to the radius of its base, or to half a mean proportional between the two axes of its elliptical base, this rule may be used without material error; but it ought not to be used with any other dimensions.

2. What is the solidity of an octagonal dome, each side of its base being 20 feet, and its height 26 feet? *Ans.* 33477·09456 feet.

3. What is the solidity of the stone work of an elliptical dome, the two diameters of its base being 40 and 30 feet, the height 17·32 feet, and the stone work in every part 4 feet thick?

Ans. 9479·086848 cubic feet.

PROBLEM IV.

To find the superficies of a dome, having the height and dimensions of its base given.

Rule. Multiply the area of the base by 2, and the product will be the superficial content required*; or, multiply the square of the diameter of the base by 1·5708.

For an elliptical dome. Multiply the two diameters of the base together, and that product by 1·5708, the last product will be the area, sufficiently correct for practical purposes.

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* This rule, like the preceding is true only when the dome is circular, and its height equal to the radius of its base. For, the surface of a sphere is equal to the curve surface of its circumscribing cylinder; and the curve surface of a cylinder whose diameter is d and height $\frac{d}{2}$, is $3{\cdot}1416\,d \times \frac{d}{2} = 1{\cdot}5708\,d^2$; also the area of a circle whose diameter is d is $\cdot 7854\,d^2$, the double of which is $1{\cdot}5708\,d^2$ the same as the rule.

For an elliptical dome, Robertson says, "To half the diameter "at the base add the height, the sum multiplied by 1·5708 will "give the superficial content nearly."

This rule, which is evidently wrong in every case, has been copied into one of the most popular School treatises on mensuration extant. The rule may be thus corrected. *Multiply the two axes of the elliptical base together, and extract the square-root of the product, call this root the diameter of the base.* Then, to half the diameter at the base add the height, the *square* of the sum multiplied by 1·5708 will give the superficial content nearly.

Example 1. The diameter of the base of a circular dome is 40 feet, and its height 20 feet, required its concave superficies.

$40^2 \times {\cdot}7854 \times 2 = 40^2 \times 1{\cdot}5708 = 2513{\cdot}28$ feet. *Answer.*

2. The two diameters of an elliptical dome are 40 and 30 feet, and its height 17·32 feet, what is the concave surface? *Ans.* 1884·96 feet.

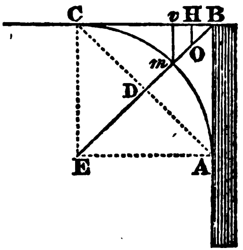
PROBLEM V.

To find the solid content of a Saloon.

Rule. Multiply the area of a transverse section by the circumference of the solid part of the saloon, taken in the middle; subtract this product from the whole vacuity of the room, supposing the walls to go upright from the spring of the arch to the flat ceiling, and the remainder will be the answer.

Example 1. What is the solid content of a saloon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide?

Here $EA = EC = EM =$ 2 feet; $\sqrt{EA^2 + EC^2} = AC = EB = \sqrt{8} = 2{\cdot}8284271$, the half of which $1{\cdot}4142136 = ED$ or CD, hence $Dm = Em - ED = 2 - 1{\cdot}4142136 = {\cdot}5857864$. Then, by rule 4, in the note page 56, the area of the segment $Amc = \frac{2}{3}(AC \times Dm) + \frac{Dm^3}{2AC} = 1{\cdot}1045696 + {\cdot}0355338 = 1{\cdot}1401$; and $\frac{1}{2}(AB \times BC) =$ 2 feet area of the triangle ABC, hence $2 - 1{\cdot}1401 = {\cdot}8598$ area of the section $ABCmA$.



Now, twice the length of the room added to twice the breadth will be the compass within the walls = 72 feet, but this will exceed the circumference of the middle of the solid part of the saloon; for $\frac{1}{2}$ (EB — E*m*) = *m*O = BO = ·4142136, and EB : BO :: BC : BH = ·29289324, hence 72 — (·29289324 × 8) = 69·65685408 the circumference of the middle of the solid part of the saloon, and 69·65685408 × ·8598 = 59·890963138 content of the solid part of the room.

Lastly, 20 × 16 = 320 area of the room, which multiplied by AB produces 640 feet the solidity of the upper part of the room, including the vacuity; then 640 — 59·890963138 = 580·109036862 feet the solidity of the vacuity or saloon.

2. If the height AB of a saloon be 3·2 feet, the chord ADC of its front 4·5 feet, the distance D*m* of its middle part from the arch 9 inches, and the mean circumference 50 feet, what is the solidity?

Ans. 138·26489 feet, content of the solid part of the room.

3. A circular building of 40 feet in diameter and 25 feet high to the ceiling, is covered with a saloon the circular arch of which is 5 feet radius; what is the solidity of the room in cubic feet?

Answer.
Solidity of the room to the spring of the arch of the saloon = 25132·8 feet
The solid work of the saloon = 637·138524 feet
The middle circumference of the solid part 119·1575334 feet
The solidity of the vacuity of the saloon = 5646·0614 feet

Whole solidity of the room = 30778·8614 feet.

PROBLEM VI.

To find the superficial content of a Saloon.

Rule. Find the breadth of the curved part of the saloon by applying a tape close to it across the surface, and its length by measuring along the middle of it entirely round the room. The product of these will be the curved surface; to which add the area of the flat ceiling, and the sum will be the whole surface.

Example 1. What is the superficial content of a saloon with a circular quadrantal arch of 2 feet radius, springing over a rectangular room of 20 feet long and 16 feet wide?

First, (by Prob. XII. page 46,) 2×90°×·0174533 = 3·1416 the length of the arch A*m*C, or breadth of the curved part of the saloon.

Secondly. By example 1st of the preceding proposition, EB = 2·8284271 and B*m* = ·8284271; now, EB : B*m* :: CB : B*v* = ·5857864.

The compass of the room within the walls = 72 feet; hence, 72 — (·5857864×8) = 67·3137088 feet, the circumference of the middle of the curved part of the saloon.

Lastly. 3·1416×67·3137088 = 211·4727475 the curved surface,

And (20—4) × (16—4) = 192· the flat ceiling.

The whole surface = 403·4727475 square feet.

2. A circular building of 40 feet in diameter, and 25 feet high to the ceiling, is covered with a saloon the circular arch of which is 5 feet radius, what will be the surface, including the windows and doors?

Answer.
Surface of the room to the spring of the arch of the saloon = 2513·28 feet
Length of the arch of the saloon = 7·854 }
Circumference of the middle of it = 112·6511 } curv. surf. = 884·76
Surface of the flat ceiling = 706·86
Total = 4104·90 feet.

THE

COMPLETE MEASURER.

GAUGING.

GAUGING is the art of measuring and finding the contents of all sorts of vessels used by maltsters, brewers, distillers, wine-merchants, &c.; such as, cisterns, coppers, vats, stills, casks, &c.

Of the SLIDING RULE.

The sliding rule is an instrument, particularly useful in gauging, made generally of box, in the form of a parallelopipedon. There are various kinds, but the most convenient, or at least that which is most used in the excise, was invented by Mr. *Verie*[*], collector of the excise.

1. The line, marked A, on the face of this rule, is called *Gunter's Line*, and is numbered 1. 2. 3. 4. 5. 6. 7. 8. 9. 10. At 2150·42 is a brass pin, marked M'B, signifying the cubic inches in a bushel of malt; at 282 is another brass pin, marked A', denoting the number of cubic inches in a gallon of ale.

2. The line marked B is on the slide, and is divided exactly in the same manner as that marked A; there is another slide B, which is used along with this; the two brass ends are then placed together, and so make a double radius numbered from the left-hand towards the right. At 231, on the second

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[*] Sliding Rules of this kind, may be had of Messrs. *Jones'*, Mathematical Instrument Makers, *Holborn*.

radius, is a brass pin marked w, signifying the cubic inches in a gallon of wine; at 314 is another brass pin, marked c, denoting the circumference of a circle whose diameter is 1. The manner of reading, and using these lines, is exactly the same as the lines A and B, on the *Carpenters Rule*, described at page 161.

3. *The back of the first radius, or slide*, marked B, contains the divisors for *ale, wine, mash-tun gallons, malt, green starch, dry starch, hard soap hot, hard soap cold, green soft soap, white soft soap, flint glass*, &c. &c. as in the table, Prob. I, following.—*The back of the second radius, or slide*, marked B, contains the gauge-points corresponding to these divisors, where s stands for squares, and c for circles.

4. The line M D on the rule, signifying malt depth, is a line of numbers beginning at 2150·42, and is numbered from the left to the right-hand 2. 10. 9. 8. 7. 6. 5. 4. 3. This line is used in malt gauging.

5. The two slides, B, just described, are always used together, either with the line A; M D; or the line D, which is on the opposite face of the rule to that already described. The line D is numbered from the left-hand towards the right, 1. 2. 3. 31. 32. which is at the right-hand end; it is then continued from the left-hand end of the other edge of the rule 32. 4. 5. 6. 7. 8. 9. 10. At 17·15 is a brass pin marked W'G, signifying the circular gauge-point for wine gallons. At 18·95 is a brass pin marked A'G for ale gallons. At 46·37, M'S signifies the square gauge-point for malt bushels. At 52·32, M'R signifies the round or circular gauge-point for malt bushels. The line D on this rule is of the same nature as the line marked D on the *Carpenters Rule*, described at page 161. The line A and the two slides B are used together, for performing multiplication, division, proportion, &c.; and the line D, and the same slides B, are used together for extracting the square root, &c.

6. The other two slides belonging to this rule are

marked C, and are divided in the same manner, and used together, like the slides B. *The back of the first radius, or slide*, marked C, is divided, next the edge, into inches, and numbered from the left-hand towards the right 1. 2. 3. 4. &c. and these inches are again divided into ten equal parts. The second line is marked *spheroid*, and is numbered from the left-hand towards the right 1. 2. 3. 4. 5. 6. 7. The third line is marked *second variety*, and is numbered 1. 2. 3. 4. 5. 6. These lines are used, with the scale of inches, for finding a mean diameter; see Prob. X. following.

The third and fourth variety are omitted on this slide, and with good reason, for it is very probable that there never was a cask made resembling either of these forms.—*The back of the second radius, or slide*, marked C, contains several factors for reducing goods of one denomination to their equivalent value in those of another. Thus | X to VI 6. | signifies that to reduce *strong* beer at 8s. per barrel, to *small* beer at 1s. 4d. you must multiply by 6. | VI. to X. ·17. | signifies that to reduce small beer at 1s. 4d. per barrel to strong beer at 8s. per barrel, you must multiply by ·17. | C 4^s to X. 27 | signifies that 27 is the multiplier for reducing cider at 4s. per barrel to strong beer at 8s. &c.

7. The two slides C, just described, are always used together, with the lines on the rule marked *Seg. St* or S S, segments standing; and *Seg. Ly* or S L, segments lying; for ullaging casks. The former of these lines is numbered 1. 2. 3. 4. 5. 6. 7. 8. which stands at the right-hand end; it then goes on from the left-hand on the other edge 8. 9. 10. &c. to 100; the latter is numbered in a similar manner, 1. 2. 3. 4. which stands at the right-hand end; it then goes on from the left hand on the other edge, 4. 5. 6. &c. to 100.

PRO-

PROBLEM I.

To find the several Multipliers, Divisors, and Gauge-points, belonging to the several Measures now used in England.

For square figures, the following multipliers and divisors are to be used.

282)1·0000(·003546 multiplier for ale gallons.
231)1·0000(·004329 multiplier for wine gallons.
268·8)1·000(·0037202 multiplier for malt gallons.
2150·42)1·000(·00046502 multiplier for malt bushels.
227)1·000(·00405 multiplier for mash-tun gallons.

So, if the solid inches in any vessel be multiplied by the said multipliers, the product will be gallons in the respective measures; or dividing by the divisors 282; 231; or 268·8; the quotient will likewise be gallons.

Note. That 282 solid inches is a gallon of ale or beer measure; 231 solid inches is a gallon of wine-measure; 268·8 solid inches is a gallon, and 2150·42 solid inches is a bushel of malt, of corn measure.

For circular areas, the following multipliers and divisors are to be used.

282)·785398(·002785 multiplier for ale gallons.
231)·785398(·003399 multiplier for wine gallons.
·785398)282·(359·05 divisor for ale gallons.
·785398)231·(294·12 divisor for wine gallons.
·785398)2150·42(2738· divisor for corn bushels.
The square root of the divisor is the gauge-point.

The gauge-point for squares in	Ale-measure, is	16·79
	Wine-measure, is	15·19
	Malt-bushel, is	46·37
The gauge-point for circular figures in	Ale-measure, is	18·95
	Wine-measure, is	17·15
	Malt-bushel, is	52·32

And thus the numbers in the following table were calculated.

A TABLE of Multipliers, Divisors, and Gauge-points, for Squares and Circles.

Note. *The Areas, &c. are all in Inches.*	Multipliers for		Divisors for		Gauge-points for	
	Squares	Circles	Squares	Circles	Squares	Circles
The side or diameter 1	1·	·785398	1·	1·27324	1·	1·128
A superficial foot..........	006944	·005454	144·	183·34	12·	13·54
A solid foot..............	·000578	·000454	1728·	2200·16	41·57	46·91
Ale gallon	·003546	·002785	282·	359·05	16·79	18·95
Wine gallon..............	·004329	·003399	231·	294·12	15·19	17·15
Malt. or corn bushel	·000465	·000365	2150·42	2738·	46·37	52·32
Malt gallon	·003720	·002922	268·8	342·24	16·39	18·5
Mash-tun gallon	·004405	·00346	227·	289·	15·1	17·07
A pound of hard soap cold ..	·036845	·028939	27·14	34·56	5·21	5·88
A pound of hot soap	·035714	·028050	28.	35·65	5·29	5·97
A pound of green soap	·038956	·0306	25·67	32·68	5·06	5·72
A pound of white soft soap	·039123	·030731	25·56	32·54	5·05	5·7
A pound of tallow net	·031844	·025101	31·4	39·98	5·6	6·32
A pound of green starch....	·028736	·022565	34·8	44·32	5·9	6·66
A pound of dry starch	·024813	·019491	40·3	51·3	6·35	7·16
A pound of flint glass	·094697	·074405	10·56	13·44	3·25	3·69
A pound of white glass	·071123	·05586	14·06	17·9	3·74	4·32
A pound of green glass	·082102	·064516	12·18	15·5	3·48	3·94

Note. It often happens in the practice of gauging, that when the one given number is set to the gauge-point on the sliding rule, the other given number will fall off the rule; hence in many cases it will be necessary to find a *second*, or *new* gauge-point. The *second* gauge-points are the square roots of 10 times the divisors in the above table. Thus for squares, the *new* gauge-point for ale is 53·10, for wine 48·06, for malt-bushels 14·66; and for circles, the *new* gauge-point for ale is 59·92, for wine 54·22, and for malt-bushels 16·54.

By the Sliding Rule.

Set 1 on B, to the old gauge-point on D, and against the other 1 on B, is the new gauge-point on D.

PROBLEM II.

To find the Area in Ale or Wine Gallons, of any rectilineal plane Figure.

RULE. Find the area in inches, by the rules given in MENSURATION OF SUPERFICIES, which divide by the proper divisor, or multiply by the proper multiplier, and the result will be the area in gallons.

Note. The areas of all plane figures, in gauging, are expressed in gallons; because there will be the same number of solid inches in any vessel of one inch deep, as there are superficial inches in its base; so that what is called by gaugers a surface, or area, is in reality a solid of one inch in depth.

Example 1. Suppose a back or cooler in the form of a parallelogram, to be 250 inches in length, and 84·5 inches in breadth; what is the area in ale or wine gallons?

Multiply 250 by 84·5, and the product is 21125, the area in inches, which divide by 282, and the quotient is 74·9 gallons of ale; or multiply by ·003546, the product is 74·90925 gallons, nearly the same; and if 21125 be divided by 231, or multiplied by ·004329, it will give 91·45 gallons of wine.

By the Sliding Rule.

	on A.		on B.		on A.		on B.
As	282	:	84·5	::	250	:	74·9
As	231	:	84·5	::	250	:	91·45

2. If the side of a square be 40 inches, what is the area in ale gallons? *Ans.* 5·67 gallons.

3. The longest side of a parallelogram is 50 inches; the shortest side 30 inches; what is the area in ale gallons? *Ans.* 5·32 gallons.

4. The side of a rhombus is 40 inches, and its perpendicular breadth 37 inches; what is its area in ale gallons? *Ans.* 5·25 gallons.

5. The length of a rhomboides is 48 inches, and its perpendicular breadth 32 inches; what is its area in wine gallons? *Ans.* 6·65 gallons.

6. The base of a triangle is 60 inches, and its perpendicular 23·5 inches; required its area in ale gallons? *Ans.* 2·5 gallons.

7. The diagonal of a trapezium is 60 inches, and the two perpendiculars from the opposite angles 15 and 27 inches; what is its area in ale gallons? *Ans.* 4·47 gallons.

8. The side of a pentagon is 50 inches, what is its area in ale gallons? *Ans.* 15·25 gallons.

9. The side of a hexagon is 64 inches, and the perpendicular from the centre to the middle of one of the sides 55·42 inches; required its area in ale, wine gallons, and malt-bushels?

Ans. { 37·73 ale gallons.
46·06 wine gallons.
4·94 malt bushels.

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area of it in Ale or Wine Gallons, &c.

Rule. Multiply the square of the diameter by ·002785 for ale, or by ·003399 for wine; or divide the square of the diameter by 359·05 for ale, or by 294·12 for wine, the products or quotients will be the respective ale and wine gallons.

For any other denomination, use the proper multiplier or divisor in the table.

Example 1. Suppose the diameter of a circle be 32·6 inches; what will be the area in ale and wine gallons?

The square of 32·6 is 1062·76.
Then 1062·76 × ·002785 = 2·9598 ale gallons.
And 1062·76 × ·0034 = 3·6133 wine gallons.
Or 359·05)1062·76(2·9599 area in ale gallons.
And 294·12)1062·76(3·6133 area in wine gallons.

By the Sliding Rule.

	on D.		on B.		on D.		on B.
As	18·95	:	1	::	32·6	:	2·96.
As	17·15	:	1	::	32·6	:	3·61.

The first terms are the circular gauge-points; see the table.

2. If the diameter of a circle be 45 inches, what is its area in ale gallons? *Ans.* 5·64 gallons.

3. If the diameter of a circle be 68 inches; required its area in ale and wine gallons, and malt-bushels?

Ans. 12·87 ale gallons.
15·72 wine gallons.
1·68 malt bushels.

PROBLEM IV.

The transverse, or longer Diameter, and the conjugate, or shorter Diameter, of an Ellipsis being given, to find its Area in Ale or Wine Gallons.

RULE. Multiply the product of the two diameters, by ·002785 for ale, by ·0034 for wine; or divide the product of the two diameters by 359·05 for ale, or by 294·12 for wine; the products or quotients will be the respective ale and wine gallons required:

quired: and for any other denomination, use the proper divisor or multiplier.

Example 1. Suppose the longer diameter of an ellipsis to be 81·4 inches, and the shorter diameter 54·6 inches; what will be the area in ale and wine gallons?

Multiply 81·4 by 54·6, and the product is 4444·44; then,

4444·44 × ·002785 = 12·38 ale gallons.
And 4444·44 × ·0034 = 15·11 wine gallons.
Or, 359·05)4444·44(12·38 area in ale gallons.
294·12)4444·44(15·11 area in wine gallons.

By the Sliding Rule.

	on A.		on B.		on A.		on B.
As	359	:	81·4	::	54·6	:	12·4 ale gallons.
As	294	:	81·4	::	54·6	:	15·1 wine gallons.

2. The transverse diameter of an ellipsis is 72 inches, conjugate 50 inches; what is its area in ale gallons? *Ans.* 10 gallons.

3. If the transverse diameter of an ellipsis be 70 inches, conjugate 50 inches; what is its area in ale and wine gallons, and malt-bushels?

Ans. { 9·74 ale gallons.
11·90 wine gallons.
1·27 malt bushels.

PROBLEM V.

To find the Content of a Prism in Ale or Wine Gallons.

Rule. Find its solid content in inches, (by Prob. III. p. 98), then divide that content in inches by 282 for ale, or by 231 for wine, the respective quotients will be the content in ale or wine gallons.

Or, find the area of the base in gallons, (by Prob. II. p. 217), and multiply that area by the vessel's depth within, the product will be its content in gallons.

Example.1. A vessel, whose base is a right-angled parallelogram, is 49·3 inches in length, the breadth is 36·5 inches, and the depth 42·6 inches; the contents in ale and wine gallons are required?

The length, breadth, and depth, being multiplied together, produce 76656·57; which, divided by 282, the quotient is 271·83 ale gallons: and divided by 231, the quotient is 331·84 wine gallons; and by dividing by 2150·42, such a cistern will be found to hold 35·65 bushels of corn.

By the Sliding Rule.

$$\overset{\text{on B.}}{\text{As the length}} \;:\; \overset{\text{on D.}}{\text{Length}} \;::\; \overset{\text{on B.}}{\text{Breadth}} \;:\; \overset{\text{on D.}}{\text{Mean proportion}}$$

$$\overset{\text{on D.}}{\text{As squ. gauge-point}} \;:\; \overset{\text{on B.}}{\text{Depth}} \;::\; \overset{\text{on D.}}{\text{Mean proportion}} \;:\; \overset{\text{on B.}}{\text{Content}}$$

$$\text{Viz. } \overset{\text{on B.}}{49{\cdot}3} \;:\; \overset{\text{on D.}}{49{\cdot}3} \;::\; \overset{\text{on B.}}{36{\cdot}5} \;:\; \overset{\text{on D.}}{42{\cdot}42}$$

$$\overset{\text{on D.}}{\left.\begin{array}{r}16{\cdot}79\\15{\cdot}19\\46{\cdot}37\end{array}\right\}} \;:\; \overset{\text{on B.}}{42{\cdot}6} \;::\; \overset{\text{on D.}}{42{\cdot}42} \;:\; \overset{\text{on B.}}{\left\{\begin{array}{l}271{\cdot}83 \text{ ale gallons.}\\331{\cdot}84 \text{ wine gallons.}\\35{\cdot}65 \text{ malt bushels.}\end{array}\right.}$$

2. Each side of the square base of a vessel is 40 inches, and its depth 10 inches; what is the content in ale gallons? *Ans.* 56·7 gallons.

3. The length of a rectangular parallelopipedon is 72 inches, breadth 33 inches, and depth 82 inches; required the content in ale and wine gallons, and in malt bushels?

$$\textit{Ans.}\left\{\begin{array}{l}690{\cdot}89 \text{ ale gallons.}\\843{\cdot}42 \text{ wine gallons.}\\90{\cdot}61 \text{ malt bushels.}\end{array}\right.$$

4. The diameter of a cylindrical vessel is 32 inches, the internal depth 45·5 inches; required its content in ale and wine gallons, and in malt bushels?

$$\textit{Ans.}\left\{\begin{array}{l}129{\cdot}78 \text{ ale gallons.}\\158{\cdot}47 \text{ wine gallons.}\\17{\cdot}01 \text{ malt bushels.}\end{array}\right.$$

PROBLEM VI.

To find the Content of any Vessel, whereof the Ends are Squares, or Rectangles of any Dimensions.

Rule. Multiply the sum of the lengths of the two ends by the sum of their breadths, to which add the areas of the two ends; this sum multiplied by one-sixth of the depth, will give the solidity in cubic inches; which, divided by 282, 231, or 2150·42, will give the content in ale gallons, wine gallons, or malt bushels.

Example 1. The top and bottom of a vessel are parallelograms; the length of the top is 100 inches, and its breadth 70 inches; the length of the bottom 80, and its breadth 56; and the depth 42 inches; the contents in ale and wine gallons are required?

$$\left.\begin{array}{l} 180 = 100 + 80, \text{ sum of the lengths} \\ 126 = 70 + 56, \text{ sum of the breadths} \end{array}\right\} \begin{array}{l}\text{of the}\\ \text{2 ends.}\end{array}$$

22680 product.
7000 area of the greater base = 100 × 70.
4480 area of the less base = 80 × 56.

34160 sum.
7 one-sixth of the depth.

239120 solidity in cubic inches.

282)239120(847·94 ale gallons.
231)239120(1035·15 wine gallons.

By the Sliding Rule.

Find a mean proportional (83·66) between the length and breadth at the top, and a mean proportional (66·93) between the length and breadth at the bottom; the sum of these is 150·59, twice a mean proportional between the length and breadth in the middle. Then

$$
\begin{array}{ccccccc}
\text{on D.} & & \text{on B.} & & \text{on D.} & & \text{on B.} \\
 & & & \left\{\begin{array}{c} :: \\ :: \\ :: \end{array}\right. & \begin{array}{r} 83{\cdot}66 \\ 66{\cdot}93 \\ 150{\cdot}59 \end{array} & \begin{array}{c} : \\ : \\ : \end{array} & \begin{array}{r} 173{\cdot}7 \\ 111{\cdot}2 \\ 563{\cdot} \end{array} \\
16{\cdot}79 & : & \frac{42}{6} & & & & \\
 & & & & & & \overline{847{\cdot}9} \text{ A G.}
\end{array}
$$

The content in wine gallons may be found by using the wine gauge-point.

2. Each side of the bottom of a vessel, in the form of the frustum of a square pyramid, is 27 inches, each side at the top 13·8 inches, and the depth 21 inches; required the content in ale gallons?

Ans. 32·07 gallons.

3. There is a tun, whose parallel ends are rectangles, the length and breadth of the top 36 and 32 inches; the length and breadth at the bottom 48 and 40 inches, and the depth 60 inches: required the content of the tun in ale gallons, wine gallons, and malt bushels?

Ans. { 323·41 ale gallons.
394·80 wine gallons.
42·4 malt bushels.

PROBLEM VII.

To find the Content of a Vessel, in the Form of the Frustum of a Cone.

Rule. To three times the product of the two diameters, add the square of their difference, multiply the sum by one-third of the depth, and divide the product by the proper circular divisor, viz. 359·05 for ale, 294·12 for wine, and 2738 for malt bushels, &c. and the quotient will be the content accordingly.

Example 1. Suppose the greater diameter 80 inches, the less diameter 71 inches, and the depth 34 inches, the content in ale or wine gallons is required.

80 greatest diameter.
71 less diameter.

9 difference.
9

81 square of the diff.

359·05)194038(540·42 A G.
294·12)194038(659·72 W G.

71 less diameter.
80 greater diameter.

5680 product.
3

17040
81 square of diff.

3)17121 *sum.*

5707
34 depth.

194038 product.

In the above example, instead of multiplying the *Sum* by one-third of the depth, one-third of the sum is multiplied by the whole depth, which amounts to the same.

By the Sliding Rule.

Find a mean proportional between the two diameters: thus,

on B. on D. on B. on D.
80 : 80 :: 71 : 75·36; again
Set 18·95 on D to 11⅓ (one-third of depth) on B.
And against 71 on D stands the content on B - 159·09
against 75·36 on D stands the content on B - 179·22
against 80 on D stands the content on B - 202·00

Sum is the whole content - 540·31

In the same manner the content in wine gallons may be found, using the wine gauge-point.

2. The greater diameter of a conical frustum is 38 inches, the less diameter 20·2, and depth 21 inches; required the content in ale gallons?

Ans. 51·07 gallons.

3. The top diameter of a conical frustum is 22 inches, bottom diameter 40 inches, and the depth 60 inches; required the content in ale gallons, wine gallons, and malt bushels?

Ans. { 165·12 ale gallons.
201·55 wine gallons.
21·63 malt bushels.

PROBLEM VIII.

To gauge and inch a Tun in the Form of the Frustum of a Cone, and to make an Allowance for the Drip or Fall.

The inching of a tun, or vessel, is finding how much liquor it will hold at every inch of its depth.

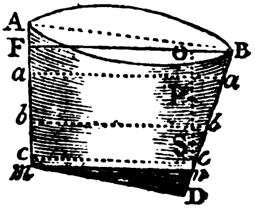
When a vessel does not stand even, or with its base parallel to the horizon, the quantity of liquor *m* D *n*, which will just cover the bottom, is called the drip or fall of the tun.

Let water be poured into the tun till the bottom is just covered, which will be when the water touches the point *m*, and suppose the measure of the water in this case to be 30·92 gallons.

Find the horizontal line F B, which will be parallel to *m n*, the surface of the liquor; the line F B will represent the surface of the liquor when the vessel is full.

Find the perpendicular depth *o n* to the surface of the liquor, which suppose to be 26 inches.

Take mean diameters parallel to F B and *m n* at every 6 or 10 inches depth. Suppose the first mean diameter *a a* at 5 inches from F B to be 83·6 inches; the second mean diameter *b b* at 15 inches from F B = 78·7 inches; and the third



mean diameter *c c* at 23 inches from F B = 74·5 inches.

Find the areas corresponding to each of these diameters, as in the third column of the following table; multiply these areas by their respective depths, and you will have the fourth column. Lastly, these contents being brought into barrels, &c. allowing 34 gallons to a barrel, and 8½ gallons to a firkin, will give the remaining columns.

Depths.	Diam.	Area.	Content	Content		
			in Gallons.	B	F.	G.
o P = 10	83·6	19·465	194·65	5	2	7·65
P S = 10	78·7	17·250	172·50	5	0	2·50
S *n* = 6	74·5	15·458	92·75	2	2	7·75
Whole 26	Cont. of the Drip.		30·92	0	3	5·42
Whole content			490·82	14	1	6·32

Now to find the content at every inch of the depth, reduce the first area 19·465 into barrels, &c. and it will be 2 F. 2·465 G. which subtracted from the whole content 14 B. 1 F. 6·32 G. leaves 13 B. 3 F. 3·8555 G. the content when 1 inch is dry; and thus continue subtracting the first area from the several remainders until 10 inches are dry, when there will remain 8 B. 2 F. 7·17 G.—Then take the second area 17·25, having first brought it into firkins, &c. and subtract it from the last remainder, and thus continue to do till you have 20 inches dry, when there will remain 3 B. 2 F. 4·67 G. —Lastly, take the third area 15·458, having first brought it into firkins, &c. and subtract it from the area when 20 inches are dry; and thus proceed till you have 26 inches dry, and there will remain, if you have made no mistake, 3 F. 5·42 G. the quantity of liquor contained in the drip or fall.

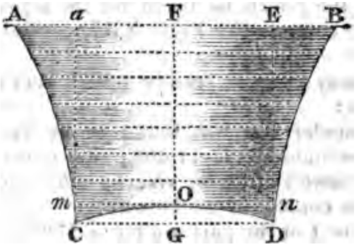
The point F may be marked as a constant dipping place, and the several contents entered into a dimension book. Then to find how much liquor there is in this fixed vessel, at any future time, take the depth of the liquor, and against that depth in your table you will find the content.

PROBLEM IX.

To gauge a Copper.

Let A B C D be a copper to be gauged.

Take a piece of small packthread, make one end fast at A, and extend the other to the opposite side of the copper at B, where make it fast. Then with some convenient instrument take the nearest distance from the deepest place in the copper, to the thread, as *a* C, or E D, which suppose to be 47 inches.



In like manner, set the end of the instrument or rule upon the top of the crown at O, and take the nearest distance to the thread, as O F, which suppose 42 inches: this subtracted from *a* C, 47, the remainder G O is 5, the height of the crown.

To find C D, the diameter of the bottom of the crown.

Measure A B, the diameter of the top, which admit to be 99 inches; then hold a thread, so that a plummet at the end thereof may hang just over C, by which means you will find the distance A *a*. Do the like on the other side; so will you find also the distance, B E, which suppose 17·5 inches each; add these two together, and subtract their sum (viz. 35) from 99, and the remainder is 64 inches, equal to C D, the diameter at the bottom of the crown. The diameter *m* O *n*, which touches the top of the crown, may be found by measuring to be 65 inches.

Now to find the content of the copper from the crown upwards, that is the part *m n* B A, the depth F O being 42 inches, you may take the diameter in the middle of every 6 inches of the depth, which suppose to be as in the second column of the following table, the numbers in the third column are the respective areas in ale gallons, found by Prob. III. the fourth column shews the content of every 6 inches; all which being added together, the sum will be the content of that part *m n* B A; that is, so much as it will hold after the crown is covered.

Now if the crown be taken for the segment of a sphere, the content (by Prob. XIII. p. 120), will be found to be 28·75 gallons.

But it may be more readily found, very near the truth, thus:

The diameter C D was found to be 64, and the area to this diameter is 11·408; this multiplied by half the crown's altitude, viz. by 2·5, gives 28·52 gallons, the content of the crown.

The content of the part *m n* D C is 57·935 gallons; from which subtract the content of the crown 28·52, and the remainder is 29·415 gallons, and so much liquor will just cover the crown.

Parts

Parts of the Depth.	Diameters.	Areas.	Content of every 6 inches.
6	95·3	25·2945	151·767
6	90·1	22·6095	135·657
6	85·0	20·1223	120·734
6	80	17·8240	106·947
6	75·2	15·7499	94·499
6	70·5	13·8426	83·056
6	66	12·1316	72·791
The sum - - - -			765·451
To just cover the crown			29·415
The whole content -			794·866

The contents in the last column may be brought into firkins and barrels, and then the content at every inch in depth, as in the VIIIth Problem, may be found and entered into the dimension book.

PROBLEM X.

To find the Content of any close Cask.

This part of gauging is the most difficult that occurs, on account of the uncertainty of the figure of the cask.

In every case, it is essential that the following dimensions be truly taken.

Viz. { The bung-diameter,
The head-diameter,
The length of the cask, } within the cask.

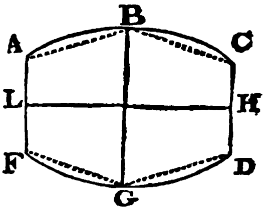
In taking these dimensions, the gauger ought carefully to observe,

1. That the bung-hole be in the middle of the cask; also, that the bung-stave, and the stave op-
posite

posite to the bung-hole, are both regular and even within.

2. That the heads of the casks are equal, and truly circular; if so, the distance between the inside of the chimb to the outside of its opposite stave, will be the head-diameter within the cask, very near.

The diameters and length of one cask may be equal to those of another, and yet one of those casks may contain several gallons more than the other.



For instance, let the figure A B C D G F represent a cask. Now, it is plain, that if the outward curve lines, A B C, and F G D, be the bounds or staves of the cask, it will hold more than if the inner dotted lines were the bounds or staves; and yet the bung diameter B G, and head diameters C D and A F, and the length L H, are the same in both those casks.

Hence it appears, that no *one* general rule can be given, by which all sorts of casks may be gauged: and therefore gaugers usually suppose every cask to be in some of these forms:

1. The middle zone or frustum of a spheroid; see page 130.
2. The middle frustum of a parabolic spindle; see page 135.
3. The lower frustums of two equal parabolic conoids; see page 133.
4. The lower frustums of two equal cones, such as the figure represented by the dotted lines above.
5. The contents of casks are sometimes found by having a mean diameter between the bung and head given; see the rule at page 139.

To find the Content of a Cask by the mean Diameter.

RULE. Multiply the difference between the head and bung-diameters, when it is less than 6 inches, by ·68 for the first variety; by ·62 for the second variety; by ·55 for the third; and by ·5 for the fourth. Or, when the difference between the head and bung-diameters exceeds 6 inches, multiply that difference by ·7 for the first variety; ·64 for the second; ·57 for the third; and ·52 for the fourth. Add this product to the head-diameter, and that sum will be a mean diameter. Square the mean diameter, and multiply that square by the length of the cask; this product multiplied, or divided, by the proper multiplier, or divisor, in the table, Prob. I. will give the content.

By the Sliding Rule.

Find the difference between the bung and head-diameter, on the inside of the slider marked C, and opposite to it, for each variety *, is a number to be added to the head diameter, for the mean diameter. Then as the gauge-point on D, is to the length on B; so is the mean diameter on D, to the content on B.

Example 1. Suppose the bung-diameter of a cask to be 32 inches, the head-diameter 24 inches, and the length 40 inches; the contents in ale gallons, for each variety, are required?

---

* It has been noticed before, that only the first and second variety of casks are placed on the rule, which is described at the beginning of gauging. The contents of casks are generally found by the sliding rule, for which reason the rules for finding the content by the pen, are given in such a manner as to agree therewith. The above methods of finding a mean diameter are not strictly true: see a dissertation on this subject in *Moss's Gauging*, Sect. X.

1. For the *spheroid*, or first variety.

Bung-diam.	32	mean diam. 29·6
Head-diam.	24	29·6
Difference	8	1776
Multiplier	·7	2664
		592
	5·6	
Head-diam.	24·	876·16 square.
		40 length.
Mean diam.	29·6	
		359·05)35046.40(97·6 gallons.
		273190
		218550
		3120

In a similar manner you will find the content for the second variety to be 94·46 ale gallons; for the third variety 90·87 ale gallons; and for the fourth variety 88·34 ale gallons.

By the Sliding Rule.

For the first variety, against 8 (the difference between the bung and head-diameter) in the line of inches, you will find 5·6 on the line marked *spheroid*, which, added to the head-diameter 24, gives the mean diameter 29·6. Then

on D.		on B.		on D.		on B.
18·95	:	40	::	29·6 mean diam.	:	97·6.

In the same manner the content of the other varieties may be found, by using their respective mean diameters, without removing the slider.

9. Suppose the bung-diameter of a cask to be 26·5 inches, head-diameter 23 inches, and length 28·3 inches; the content in ale gallons, for each variety, is required?

Ans. 50·8 for the first variety.
50·0 for the second variety.
48·8 for the third variety.
48·28 for the fourth variety.

To find the Content of ANY *Cask.*

GENERAL RULE.

Add into one sum
39 times the square of the bung-diameter,
25 times the square of the head-diameter, and
26 times the product of those diameters;
multiply the sum by the length of the cask, and the product by the number ·00034; then this last product, divided by 9, will give the wine gallons; and, divided by 11, will give the ale gallons.

Dr. *Hutton's Mathematical Dict.* vol. i. p. 528.

A General TABLE for finding the CONTENT of of *any* Cask by the SLIDING RULE.

Q.	W.G.	A.G.	Q.	W.G.	A.G	Q.	W.G.	A.G.
·50	21·31	23·55	·67	19·78	21·95	·84	18·36	20·29
·51	21·22	23·45	·68	19·69	21·76	·85	18·28	20·20
·52	21·13	23·35	·69	19·60	21·66	·86	18·20	20·12
·53	21·04	23·24	·70	19·51	21·56	·87	18·13	20·03
·54	20·95	23·14	·71	19·43	21·47	·88	18·05	19·95
·55	20·85	23·04	72	19·35	21·37	·89	17·97	19·86
·56	20·76	22·94	·73	19·26	21·28	·90	17·89	19·77
·57	20·67	22·84	·74	19·18	21·18	·91	17·81	19·68
·58	20·57	22·74	·75	19·09	21·09	·92	17·74	19·60
·59	20·48	22·64	·76	19·01	21·01	·93	17·67	19·52
·60	20·39	22·54	·77	18·93	20·92	·94	17·59	19·44
·61	20·30	22·45	·78	18·85	20·83	·95	17·51	19·36
·62	20·21	22·35	·79	18·77	20·74	·96	17·44	19·27
·63	20·13	22·25	·80	18·69	20·65	·97	17·37	19·18
·64	20·04	22·15	·81	18·61	20·56	·98	17·30	19·10
·65	19·95	22·05	·82	18·53	20·47	·99	17·22	19·02
·66	19·87	21·86	·83	18·44	20·38	1·00	17·15	18·95

The above table of gauge-points was calculated by Mr. *John Lowry*, of the Royal Military College, *Sandhurst*. See the *Mathematical* and *Philosophical Repository*, page 119, &c.

The Use of the TABLE.

Divide the head-diameter by the bung-diameter, to two places of decimals; find the quotient in the column marked Q, and against it stand the wine and ale gauge-points. Then,

As the gauge-point on D : the length of the cask on B :: the bung-diameter on D : the content on B.

Example 1. The head-diameter of a cask is 34·8 inches, the bung-diameter 44·8 inches, and the length of the cask 54 inches; required the content in ale and wine gallons?

Bung-diam. 44·8
44·8

3584
1792
1792

200704
39

1806336
602112

78274·56
30276·
40535·04

149085·6 Sum.
54 Length.

5963424
7454280

8050622·4

Head-diam. 34·8
34·8

2784
1392
1044

1211·04
25

605520
242208

30276·00

Bung-diam. 44·8
Head-diam. 34·8

3584
1792
1344

Prod. 1559·04
26

935424
311808

40535·04

8050622·4
·00034

322024896
241518672

11)2737·211616

248·8374 Ale gallons.

304·1346 Wine gallons.

By the Sliding Rule.

The quotient arising by dividing the head-diameter by the bung-diameter is ·77, and the wine and ale gauge-points are 18·93 and 20·92.

$\left.\begin{matrix}18{\cdot}93\\20{\cdot}92\end{matrix}\right\}$ on D : 54 on B :: 44·8 on D : $\left\{\begin{matrix}304{\cdot}\ \text{W G.}\\248{\cdot}\ \text{A G.}\end{matrix}\right\}$ on B.

2. The head-diameter of a cask is 24·5 inches, bung-diameter 31·5, and length 42 inches; required its content in ale and wine gallons?

Ans. $\left\{\begin{matrix}95{\cdot}766\ \text{ale gallons.}\\117{\cdot}047\ \text{wine gallons.}\end{matrix}\right.$

The content by the sliding rule is exactly the same.

3. Required the content of a cask in ale and wine gallons, whose head-diameter is 24 inches, bung-diameter 32 inches, and length 40 inches?

Ans. $\begin{cases} 112 \cdot 28 \text{ wine gallons.} \\ 91 \cdot 86 \text{ ale gallons.} \end{cases}$

The content by the sliding rule is the same.

4. The bung-diameter of a cask is 48 inches, head-diameter 35·8 inches, and the length 55 inches; required the content in ale and wine gallons?

Ans. $\begin{cases} 283 \cdot 178 \text{ ale gallons.} \\ 346 \cdot 106 \text{ wine gallons.} \end{cases}$

The content is the same by the sliding rule.

5. The head-diameter of a cask is 28·2 inches, bung-diameter 33·8 inches, and length 48 inches; required the content in ale and wine gallons?

Ans. $\begin{cases} 132 \cdot 367 \text{ ale gallons.} \\ 161 \cdot 782 \text{ wine gallons.} \end{cases}$

The content by the sliding rule is the same.

PROBLEM XI.

Of the ullage of Casks.

The ullage of a cask is what it contains when only partly filled, and is considered in two positions, *viz.* standing on its end, or lying on one side.

To ullage a lying Cask.

Rule. Divide the wet inches by the bung-diameter; find the quotient in the column *height*, (in the table at the end of the book,) take out the corresponding *Area seg.*; multiply this area by the content of the cask, and that product by 1·27324; or divide it by ·7854, the last product, or quotient, will give the ullage nearly.

By

By the Sliding Rule.

Set the bung-diameter on C to 100 on the line marked *Seg. Ly* or S L, *viz.* segments lying: Then look for the wet inches on C, and observe what number stands against it on the segments, which call a fourth number. Then, set 100 on A to the content of the cask upon B, and against the fourth number, before found on A is the quantity of liquor in the caſk on B.

Example 1. Suppoſing the bung-diameter of a lying caſk to be 32 inches, its content 97·6 ale gallons; required the ullage for 19 wet inches?

1·000 whole diam.
32)19·000(·594 height. ·7854 whole area.
·406 height. *Area seg.* ·299255
·486145 rem.
The content 97·6
2916870
3403015
4375305
·7854)47·4477520(60·4 gal.

Note. Because, in this example, the quotient of the wet inches, divided by the bung-diameter, exceeds the heights in the table, it is necessary to subtract it from the whole diameter, &c.

By the Sliding Rule.

	on C.		on S L.		on C.		on B.	
As	32	:	100	::	19	:	62·5	fourth number.
	on A.		on B.		on A.		on S. L.	
As	100	:	97·6	::	62·5	:	61	gallons. *Ans.*

2. The bung-diameter of a lying cask is 25 inches, the content 48·3 ale gallons, and the wet

inches 11; required the quantity of liquor in the cask?

Ans. { 20·46 by calculation.
19·9 by the sliding rule.

3. The bung-diameter of a lying cask is 32 inches, its content 91·6 gallons, and the wet inches 8; required the quantity of liquor in the cask?

Ans. { 17·9 gallons, by calculation.
16· gallons, by the sliding rule.

To ullage a standing Cask.

Rule. Multiply the difference between the squares of the bung and head diameters, by the square of the distance of the liquor's surface from the middle of the cask; and divide the product by the square of half the length of the cask, subtract one-third of the quotient from the square of the bung-diameter, and multiply the remainder by the distance of the liquor's surface from the middle of the cask.

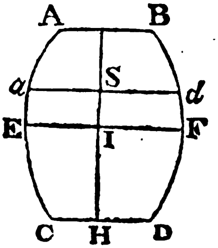
The last product divided by 359·05 for ale, or 294·12 for wine, will give the quantity of liquor above, or under half the content of the cask, according as the wet inches exceed, or fall short of half the length of the cask.

By the Sliding Rule.

Set the length of the cask on C to 100 on the line marked *Seg. St.* or SS *viz.* segments standing: Then, look for the wet inches on C, and observe what number stands against it on the segments, which call a fourth number.

Then, set 100 on A to the content of the cask upon B, and against the fourth number, before found, A is the quantity of liquor in the cask on B.

Example 1. Suppose in the annexed cask, whose content is 97·6 ale gallons, that the bung-diameter EF be 32 inches, head-diameter AB 24 inches, length 40 inches, and the wet inches SH 26; what quantity of ale is contained in the cask?



32 = EF
32
64
96
1024
576
448 Diff.
36
2688
1344
400)16128
3)40·32 Quot.
13·44

24 = AB
24
96
48
576

26 = SH
20 = IH
6 Diff. = SI
6
36 Square of SI

20 = IH
20
400 = IH^2

1024 Sqaare of EF
13·44
1010·56
6 = SI

359·05)6063·36(16·887 gallons, above half the content of the cask, *viz.* in *a d* EF. To which add 48·8 gallons, half the content of the cask, and the sum is 65·687, the quantity of liquor in the cask.

By the Sliding Rule.

on C. on SS. on C. on SS.
As 40 : 100 :: 26 : 66·1 fourth number.

on A. on B. on A.
As 100 : 97·6 :: 66·1 : 64·6 gall. *Ans.*

2. The bung-diameter of a standing cask is 35 inches, head-diameter 28·7, length 40, wet inches 30; content in ale gallons 121·5, in wine gallons 148·5; required the content of the ullage in ale and wine gallons?

Ans. { 93·93 ale gallons.
114·76 wine gallons.

The answer by the sliding rule is the same.

3. The bung-diameter of a standing cask is 26·5 inches, head-diameter 23 inches, length 28·3, wet inches 11; and the content in ale gallons, 48·3; required the ullage?

Ans. { 18·01 gallons by calculation.
18·3 by the sliding rule.

THE
COMPLETE MEASURER.

LAND-SURVEYING.

LAND-surveying is the art of measuring, planning, and finding the superficial content, of any field, or parcel of land. In this kind of measuring, the area or superficial content is always expressed in acres, or acres, roods, and perches; and the lengths of all lines, in the field, or parcel of land, are measured with a chain such as is described at pages 27 and 28.

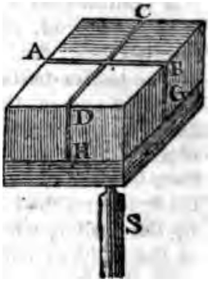
A line, or distance on the ground, is thus measured.—Having procured 10 small arrows, or iron rods, to stick in the ground at the end of each chain; also some station-staves, or long poles with coloured flags, to set up at the end of a station line, or in the angles of a field; two persons take hold of the chain, one at each end, the foremost, for the sake of distinction, is called the leader, the hindermost, the following.

A station-staff is set up in the direction of the line to be measured, if there be not some object, as a tree, a house, &c. in that direction.

The leader takes the 10 arrows in his left hand, and one end of the chain, by the ring, in his right hand, and proceeds towards the station staff, or other object. The follower stands at the beginning of the line, holding the other end of the chain, by the ring, till it is stretched straight, and laid, or held level, by the leader, whom he directs, by waving his hand to the right or left, till he see him exactly in a line with the object towards which they are measuring.

The leader then sticks an arrow upright in the ground, as a mark for the follower to come to, and proceeds forward another chain, at the end of which he is directed, as before, by the follower; or he may now, and at the end of every other chain, direct himself, by moving to the right or left, till the follower and the object measured from, be in one straight line. Having stuck down an arrow, as before, the follower takes up the arrow which the leader first stuck down. And thus they proceed till all the ten arrows are employed, or in the hands of the follower, and the leader, without an arrow, is arrived at the end of the eleventh chain length. The follower then sends or carries the ten arrows to the leader, who puts one of them down at his end of the chain, and proceeds with the other nine and the chain, as before. The arrows are thus changed from the one to the other, till the whole line is finished, if it exceed ten chains; and the number of changes shews how many times ten chains the line contains. Thus, if the whole line measure 36 chains 45 links, or 3645 links, the arrows have been changed three times, the follower will have 5 arrows in his hand, the leader 4, and it will be 45 links from the last arrow, to be taken up by the follower, to the end of the line.

Of the Surveying Cross, or Cross-staff.



The surveying cross consists of two pair of sights at right angles to each other: these sights are sometimes pierced out in the circumference of a thick tube of brass; and sometimes the cross-staff consists of four sights strongly fixed upon a brass cross, and when used is screwed on a staff, having a sharp point to stick in the ground. The accuracy of the cross-staff depends on the sights being exactly at right angles to each other.—A cross-staff may be easily made by a carpenter: thus, take a piece of beech or box, A D B C, of four or five inches in breadth, and three or four inches in depth, and upon A D B C draw two lines, A B and C D, crossing each other at right angles. Then, with a fine saw, make two slits, A B G and C D H, of about two inches in depth; fix this piece of wood upon a staff S, of about 4 and a half, or 5 feet in length, pointed at one end, so that it may easily stick into the ground.

PROBLEM I.

To measure Off-sets with a Chain and Cross-staff.

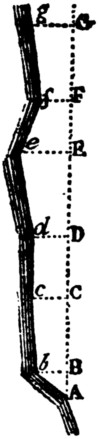
Let A *b c d e f g* be a crooked hedge, river, or brook, &c. and A G a base line. First, begin at the point A, and measure towards G; when you come to B, where you judge a perpendicular must be erected, place the cross staff in the line A G in such a position that both G and A may be seen through two of the sights, looking forward towards G, and backward towards A. Then look along one of the cross sights, and if it point directly to the corner, or bend at *b*, the cross-staff is placed right; otherwise move backward or forward along A G till the cross sights do point to *b*, and measure B *b*, which set down in links: proceed thus till you have taken all the off-sets, as in the following field book *.

---

* A field book is usually divided into three columns. The middle column contains the different distances on the chain-line, and in the right and left-hand columns, the off-sets and remarks are entered. The best method of entering field-notes, though perhaps not the most convenient, is to begin at the bottom of the page, and write towards the top.

FIELD-BOOK.

Off-sets Left.	Base Line A G, or, ⊙, Station.	Off-sets Right.
91	785 = A G	—
57	634	—
98	510	—
70	340	—
84	220	—
62	45	—
—	⊙ A go North.	—



To lay down the Plan.

Draw the line A G of an indefinite length : then by a diagonal scale, such as described at page 25, set off A B equal to 45 links, draw B *b* perpendicular to A G, and equal to 62 links. Next set off A C equal to 220 links, or 2 chains 20 links; draw C *c* perpendicular to A G, and equal to 84 links: then set off A D equal to 340 links, or 3 chains 40 links, and make D *d* equal to 70 links : proceed thus till you have completed the figure.

To cast up the Content.

A B *b* must be measured as a triangle ; B C *c b*, C D *d c*, D E *e d*, &c. must be measured as trapezoids; see page 37. Some authors direct you to add all the perpendiculars B *b*, C *c*, &c. together, and divide their sum by the number of them, then multiply the quotient by the length A G ; but this method is always erroneous except the off-sets B *b*, C *c*, &c. be equally distant from each other.

CALCU-

CALCULATION.

AB = 45	AC = 220	AD = 340	AE = 510	AF = 634	AG = 784
B*b* = 62	AB = 45	AC = 220	AD = 340	AE = 510	AF = 634
90	BC = 175	CD = 120	DE = 170	EF = 124	GF = 151
270					
	B*b* = 62	C*c* = 84	D*d* = 70	E*e* = 98	F*f* = 57
2790	C*c* = 84	D*d* = 70	E*e* = 98	F*f* = 57	G*g* = 91
	Sum 146	Sum 154	Sum 168	Sum 155	Sum 148
	B*c* = 175	CD = 120	DE = 170	EF = 124	FG = 151
Prod.	25550	1 480	28560	19220	22348

2790 = double area of A B *b*.
25550 = double area of B C *c b*.
18480 = double area of C D *d c*.
28560 = double area of D E *e d*.
19220 = double area of E F *f e*.
22348 = double area of F G *g f*.

2)116948 = double area of the whole in squ. links.

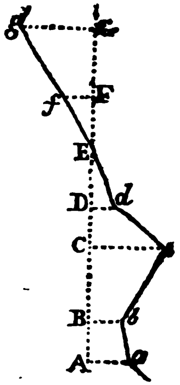
58474 = area in square links.

·58474 = area in acres = 0 A. 2 R. 13·5584 P.

2. Required the plan and content of part of a field, from the following field-book.

FIELD-BOOK.

Off-sets Left.	Base Line A G.	Off-sets Right.
150	700 = A G	—
60	550	—
—	450	Cross Hedge.
—	325	46
—	250	150
—	100	60
	⊙ A go North.	100



The figure must be laid down, and the content calculated as in the first example. Thus you will find the area of the part E*dcba*A to be 1 R. 13·4 P. and of the part E*fg*G to be 30 perches; so that the whole is 2 roods 3·4 perches.

PROBLEM II.

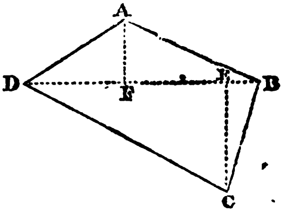
To measure a Field in the Form of a Trapezium.

Set up station-staves, or long poles, at the corners A, B, C. Then begin at D, and measure along the diagonal D B, in a straight line, till you come to the place of the perpendicular A F, which will be known by looking backward and forward through the sights of the cross-staff, as before directed. Make a mark at F, and measure the perpendicular A F; then proceed from F towards B, and when you come to E, the place where the second perpendicular will fall, make a mark, and measure the perpendicular E C: lastly, continue your measure from E to B. You may either draw a rough plan of the field by the eye, and write the length of the diagonal and perpendiculars on it, which some writers recommend as the best method; or set them down in a field-book, *thus:*

FIELD-BOOK.

—	1360 = A B.	—
—	1190	625
342	600	—
—	⊙ D go East	—
Off-sets Left.	Station ⊙, or Base Line.	Off-sets Right.

To lay down the Plan.



Draw the station-line D B equal to 1360 links, or 13·60 chains; from D set off D F equal to 600 links, or 6 chains; draw A F perpendicular to D B, and equal to 342 links, or 3 chains 42 links: make D E equal to 1190 links, and at E erect the perpendicular E C equal to 625 links. Join D A, A B, B C, and D C, and the field is constructed.

CALCULATION.

This field being a trapezium, its content must be found as directed in Prob. VII. p. 35.

A F = 342 links.
E C = 625.

Sum 967
680 half the diagonal D B.

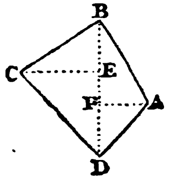
77360
5802

6·57560 area in acres = 6 acres 2 roods 12·096 perches.

2. Required the plan and content of a field from the following field-book.

FIELD-BOOK.

Off-sets Left.	Station ⊙, or Base Line.	Off sets Right.
—	592 = D B	—
306	362	—
—	214	210
—	⊙ D go North.	—



Ans. The content is 1 acre 2 roods 4·3776 perches.

Note. In either of the foregoing figures, if the sides D C; C B, B A, A D, and the diagonal D B, had been measured with the chain, the figures might have been planned, and their contents found by Prob. V. p. 31, without using a cross-staff, or measuring the perpendiculars A F and C E.

Unskilful measurers frequently measure *round* a four-sided field, and find the content by adding each two opposite sides together, and taking half their sum; then they multiply these half sums together for the area. It may be necessary to inform the learner, that this method is, generally, very erroneous, and ought *never* to be practised.

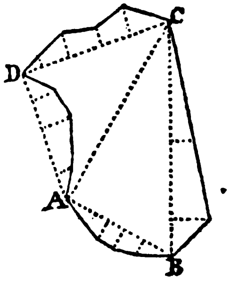
PROBLEM III.

To measure a four-sided Field with crooked Hedges.

Set up staves, or poles, at the corners D, C, B, as before directed. Then begin at A, and measure from A to B, noting all the necessary off-sets, and in this manner go round the field, then measure the diagonal A C.

FIELD-BOOK.

Diagonal A C = 1294.		
—	800 = D A	—
100	700	—
200	400	—
150	200	—
—	⊙ D go Southward.	—
—	1000 = C D	—
—	900	50
—	700	150
—	500	100
—	300	200
—	⊙ C go Westward.	—
—	1500 = B C	—
—	700	150
—	200	200
—	⊙ B go North.	—
—	750 = A B	—
—	550	80
—	400	130
—	300	100
—	⊙ A go S.E.	—
Off-sets Left.	Stations, or Base Lines.	Off-sets Right.



CALCULATION.

	Acres.
The sides of the triangle A B C are 750, 1500, and 1294 links, and its content, by Prob. V. page 31.	4·85239
The sides of the triangle A C D are 1600, 800, and 1294 links, and its content, by Prob. V. page 31.	3·99907
Content of the off sets along A B =	0·50250
Ditto ——————— along B C =	1·67500
Ditto ——————— along C D =	1·07500
Sum =	12·10396
Content of the off-sets along D A, deduct,	1·00000
Remains the content of the field =	11·10396

PROBLEM IV.

To measure an irregular Field.

Set up station staves at every angle, and divide the field into trapeziums and triangles;

Thus, begin at A and measure in a straight line towards C; at *a* take the perpendicular *a* I, and at *m* the perpendicular *m* B. Then measure from C towards H; at *n* take the perpendicular *n* D, and at *o* the perpendicular *o* I.

Next measure from H towards E; at *w* take the perpendicular *w* G and at *v* the perpendicular *v* D.

Lastly, measure from E towards G; and at *s* take the perpendicular *s* F*.

To cast up the content, draw a rough figure of the field, and place the dimensions upon the lines as you proceed.

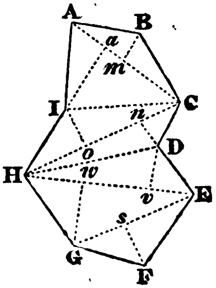
Or, the dimensions being given in the following field-book, required the figure and the content.

---

* If any of the hedges A B, B C, &c. be crooked, these lines must be measured and off-sets taken as in Problem III.

FIELD-BOOK.

—	900 = E G	—
268	550	—
—	⊙ E, go S. W.	—
—	1100 = H E	—
280	790	—
—	350	410
—	⊙ H, go East	—
—	1180 = C H	—
—	710	280
140	350	—
—	⊙ C, go S. W.	—
—	900 = A C	—
200	430	—
—	300	450
—	⊙ A, go S. E.	—
Off-sets Left.	Stations, ⊙, or Base Lines.	Off-sets Right.



CALCULATION.

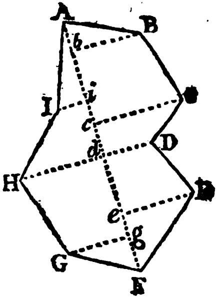
	Square Links.
The area of the trapezium A B C I =	292500
The area of the trapezium C D H I =	247800
The area of the trapezium H G E D =	379500
The area of the triangle E F G =	120600
The area of the whole -	1040400

Cut off five figures from the right-hand, and the result will be 10·40400 acres = 10 acres, 1 R. 24·64 P.

2. Required the plan and content of an irregular field, from the following

FIELD-BOOK.

Off-sets Left.	Station, ⊙, or Base Line.	Off-sets Right.
—	1620 = F A	—
—	1410	435
140	1100	—
—	940	600
553	735	265
—	380	480
380	240	—
—	⊙ F, go N. W.	—



This field when constructed, is exactly similar to the former one. The content of the off-sets must be found as in Problem I. Thus,

	Square Links.
The content of A B C D E F will be found	601000
The content of A I H G F	439390
The area of the whole -	1040390

Cut off five figures, and the result will be 10·40390 acres = 10 A. 1 R. 24·624 P.

PROBLEM V.

*To survey a large Estate**.

If the estate be very large and contain a great number of fields; it cannot be accurately surveyed and planned, by measuring each field separately, and then putting them together; nor by taking all the angles, and measuring the boundaries which enclose it. For, in these cases, the several small errors will be so multiplied, that the different fields cannot be correctly laid down in the plan, and the whole figure of the estate will be distorted.—Proceed in the following manner.

---

* *Emerson's* Surveying, page 47, et seq.

1. Walk over the estate two or three times in order to get a perfect idea of it, or till you can retain the figure of it in your mind. And to assist your memory make a rough draft of it on paper, putting the names of the different fields within them, and noting down the principal objects.

2. Choose two or more elevated places in the estate, for stations, from which all the principal parts of it can be seen; and let these stations be as far distant from each other as possible. In selecting the stations, be careful that the lines which connect them may run along the boundaries of the estate, or some of the hedges to which off-sets may be taken when necessary. The smaller the number of stations you have to command the whole, the more exact your work will be, and especially if two or more lines proceed from one station.

3. Take such angles, between the stations, as you think necessary, and measure the distances from station to station, always in a straight line: these things must be done, till you get as many angles and lines as are sufficient for determining all the points of the stations. In measuring the several station-distances, mark, accurately, where you cross any hedges, ditches, roads, lanes, paths, rivulets, &c. and where any remarkable object is placed, by measuring its perpendicular distance from the station-line; and be particularly careful always to measure in a straight line, which you may determine by taking backsights and foresights along the station line with a cross-staff or a theodolite. Also, take off-sets to all the ends of hedges, and to the different bends thereof which may fall near the station-line, and to any pond, house, mill, bridge, &c.

noting every thing down that is remarkable; for these are the *data*, by which the places of such objects are to be determined on your plan.

Be careful to set marks up at the intersections of all hedges with the station-line, that you may know where to measure from when you come to survey the particular fields which are crossed by this line. These fields must be measured as soon as you have completed your station line, whilst they are fresh in your memory. In this manner all the station-lines must be measured, and the situations of all the adjacent objects determined. It will be proper to lay down the work on paper every night, when you go home, that you may see how you go on.

4. With respect to the internal parts of the estate, they must be determined in like manner, by new station-lines: for, after the main stations are determined, and every thing adjoining to them, then the estate must be subdivided into two or three parts by new station lines; taking the inner stations at proper places, where you can have the best view. Measure these station lines as you did the first, and all their intersections with hedges, ditches, roads, &c, also take off-sets to the bends of hedges, and to such objects as appear near these lines. Then proceed to survey the adjoining fields, by taking the angles which the sides make with the station-line at the intersections, and measuring the distances to each corner from these intersections: for, every station-line will be a *basis* to all future operations; the situation of every object being entirely dependent on them, and therefore they should be taken of as great length as possible; and it is best for them to run along some of the hedges or boundaries of one or more fields, or to pass through some of their angles. All things being determined for these stations, you must take more inner stations, and continue to divide and subdivide, till at last you come to single fields; repeating the same work for

the inner stations as for the outer ones, till the whole is finished. The oftener you close your work, and the fewer lines you make use of, the less you will be liable to error.

5. An estate may be so situated that the whole cannot be surveyed together, especially if one part cannot be seen from another. In this case, you may divide it into three or four parts, and survey each part separately, as if they were lands belonging to different persons, and at last join them together.

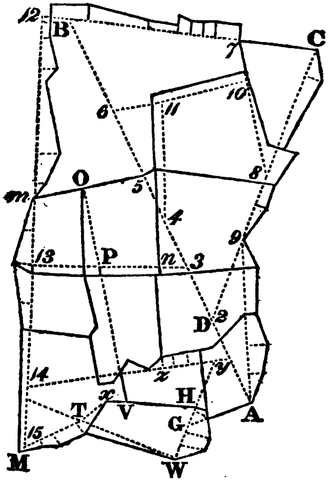
6. As it is necessary to protract or lay down the work as you proceed in it, you must have a scale of a proper length for that purpose. To construct such a scale, measure the whole length of the estate in chains, then consider how many inches long your plan must be, by these means you will be able to ascertain how many chains to an inch your scale must be, and make it accordingly, or choose one ready made of the proper dimensions. Ivory scales, *chamfered*, and graduated close to the edge, are the most convenient.

7. Then trees in every hedge-row should be placed in their proper situations, but these may be taken by guess, in a rough draft, sufficiently exact, being only for ornament; excepting such as are at remarkable places, as at the ends of hedges, at stiles, at gates, &c.; and these must be measured. In all the hedges, observe on which side the ditch is situated, and to whom the fences belong.

Example by the chain and cross-staff.

The measurement begins at A, and a direct line is measured to B, in which line every station, intended to be measured from, is carefully marked. Thus at 2* a line is measured to the right, at *3 to the left, at 4* to the right, at *5 to the left, at 6* to the right. From B the measurement continues to C (the mark ┌ shews that it goes to the right) and from C to D which completes the large triangle D B C;

the correctness of the admeasurement is proved by other station-lines, as from 8* to 7*, *10 to 6*, and *11 to 4*.



Again, C B is continued in a straight line to 12, and the measurement proceeds from 12 to 15; this line cannot be laid down on the plan till the distance from *14 to A D is measured, and the admeasurement is checked by measuring from *3 to *13, and from *5 to *m*. For the other lines see the following *Field Book*, where the stations measured *from* are inserted in the left hand corner, and the stations measured *to* in the right; where a hedge is crossed by a station-line nearly at right angles it is marked thus —, when obliquely thus ′ or thus \`, when very obliquely thus |, other appropriate marks might have been introduced, but they cannot be expressed conveniently without having types cast for the purpose.

	1794	- - - - *to* * 9
— Cross hedge	1464	Cross hedge 22 —
′ Cross hedge	920	Cross hedge 32 —
	650	60
From A	350	48
	3074	- - - - *to* D
— Cross hedge	2494	Cross hedge —
* 9 touch hedge	2072	
54	1730	
— 80	1530	Cross hedge —
	1420	8 *
56 + 30	1170	Cross hedge —
From C ┐ 40	280	
Along the hedge	2574	- - - - *to* C
50 touch hedge	1794	7 *
84	1464	
76	1328	
96	1240	
52 + 34	1130	
34	860	
From B ┌ 66	190	
	4450	- - - - *to* B
	3480	6 *
* 5	2620	
′ Cross hedge	2590	Cross hedge ′
ı Cross hedge	2210	Cross hedge |
	2080	4 *
* 3	1574	
— Cross hedge	1550	Cross hedge —
	990	D or 2 *
′ Cross hedge	806	Cross hedge ′
From A	⊙ A	go North-west.
Off-sets left.	Station Lines ⊙.	Off-sets right.

* 15	4440	86 - - - *to* M.
* 14	4420	
	3884	
— Cross hedge	3380	Cross hedge 60 —
	2992	90
\ Cross hedge	2592	Cross hedge \
* 13	2544	120
	2070	56
/ Cross hedge	1900	Cross hedge /
60	1770	
160	1320	
40	650	
80	360	
From 12 20	170	
	220	- - - - *to* 12
C B produced to 12 ⌐	190	Cross hedge 46
	1310	56 - - - *to* 4 *
	836	56
— Cross hedge	684	Cross hedge 50 —
From * 11		
	1480	- - - - *to* 6 *
— Cross hedge	1004	Cross hedge 36 —
* 11	930	
	700	48
From * 10	400	30
	1480	Along hedge *to* 7 *
— Cross hedge	1004	Touch hedge |
* 10	980	
	610	34
From 8 *	280	32
Off-sets left.	Station Lines ⊙.	Off-sets right.

— Cross

	1600	- - - *to* * 13
— Cross hedge	1028	Cross Hedge —
	940	P
44	666	
70	310	
From * 3 *to* * 13 60	236	
	2280	Hedge
	2270	- - - - *to* V
	1552	180
	1380	96
	950	110
— Cross hedge	860	Cross hedge —
From O *to* V.	768	P
	1094	- - - - *to m*
From * 5 *to m.*	644	O
480 *to* D	2148	510 *to* A ; - *to* A D
	1950	Y
— 80 Cross hedge	1836	Cross hedge —
60	1724	
60	1600	
30	1480	Z
Touch hedge	1320	
′ Cross hedge	1080	Cross hedge ′
Cross hedge	750	Cross hedge 50 —
From 14 *to* A D.		
Off-sets left.	Station Lines ⊙.	Off-sets right.

From x	360	- - - - *to* T
	954	- - - - *to x*
From H	730	V
	744	- - - - *to* G
From A	672	Cross hedge —
	1160	- - - - *to y*
\ Cross hedge	890	Cross hedge \
H Cross hedge	570	Cross hedge 40 —
	376	G
	256	150
From W	144	130
	580	- - - *to* * 15
40	500	
76	300	
From T *to* * 15 76	100	
	1068	- - - - *to z*
70 *From n* towards P	1032	Cross hedge
	1676	- - - - *to* W
	896	24 [corner
	632	Cross hedge 50 to
180 *From* * 14 towards * 15	588	T
Off-sets left.	Station Lines ⊙.	Off-sets right.

PRO-

PROBLEM VI.

To survey a Town or City[*].

Here it will be proper to have an instrument for taking angles, as a theodolite or a plain table: the latter is a very convenient instrument, because the minute parts may be drawn upon it on the spot. A chain of 50 feet long, divided into 50 links of one foot each, will be more convenient than the common surveying chain, and an offset-staff of 10 feet long will be very useful.

Begin at the meeting of two, or more, of the principal streets, through which you can have the most extensive view, and consequently the longest station-lines. Having fixed your instrument at this point, draw lines of direction along the streets, using two men as marks, or use poles fixed on wooden pedestals, or such objects as may conveniently present themselves, viz. windows, doors, corners of houses, &c.

Measure the station-lines with the chain, and take off-sets with the staff to all remarkable places on the right and left, such as churches, public buildings, markets, halls, colleges, remarkable houses, &c. and where necessary take their dimensions.

Remove the instrument to another station in one of the lines already measured, then proceed as before, and so on till the whole is finished.

The principal streets being measured, proceed to the smaller and intermediate streets; and, lastly, to the lanes, alleys, courts, yards, and every part requisite to be represented in your plan.

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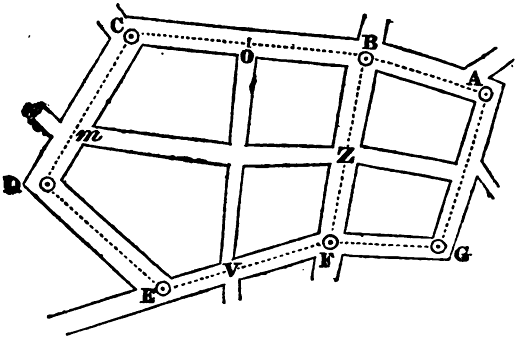
[*] *Emerson's Surveying*, p. 57, et seq.

Thus,



Thus, let B be the first station, where a street runs off to the right, take the angle C B F, and measure the length of the street B C; when you come to the street O, on the left hand, note its distance from B, and measure the width of it. Remove the instrument to C, the mark which is at F to B, and that which is at C to D, and measure the angle B C D, noting the street on the right-hand of C.

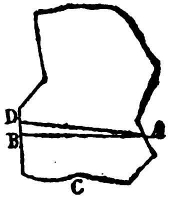
Measure C D, and be careful not to omit taking the dimensions of the streets *m* and *n* on the left and right, and also their positions. Remove the instrument to D, the mark which is at B to C, and that which is at D to E, then take the angle C D E.

Bring the mark C to D, and E to F; then fix the instrument at E, and take the angle D E F; measure D E and E F: in measuring along E F, note the two streets on the right and left at V. Again,

Place the instrument at F, and take the angle E F B; at F, a street goes forwards to G, and another turns to the right: measure F B, and in the way take the width and situation of the two streets at Z; having arrived at B, a part of your survey is completed, and may be laid down on paper. In a similar manner, you must proceed till the whole is finished.

PROBLEM VII.

To cut off from a Plan a given number of Acres, &c. by a Line drawn from any point on the side of it.



Let A be the given point in the annexed plan, from which a line is to be drawn, towards B, so as to cut off five acres 2 roods 14 perches. Draw A B, so as to cut off a quantity A B C, as near the quantity proposed as you can judge; and suppose the true quantity of A B C, when calculated to be only 4 A. 3 R. 20 P., which is 2 R. 34 P. = 114 perches = 71250 square links too little. Then measure A B, which suppose equal to 1234 links, by the half of which, *viz.* 617 links, let 71250 links be divided; the quotient, 115 links, will be the altitude of the triangle to be added, whose base is A B, therefore make B D = 115 links, and draw A D, which will cut off the quantity required.

MISCELLANEOUS QUESTIONS IN MENSURATION.

1. If a pavement be 47 feet 9 inches long, and 18 feet 6 inches broad, how many yards are contained in it? *Ans.* 98 yards 1 foot.

2. There is a room whose length is 21·5 feet, and the breadth 17·5 feet, which is to be paved with stones, each 15 inches square; how many such stones will pave it? *Ans.* 167$\frac{2}{5}$ stones.

3. There is a room 109 feet 9 inches about, and 9 feet 3 inches high, which is all (except two windows, each 6 feet 6 inches high, and 5 feet 9 inches broad) to be hung with tapestry that is ell-broad; how many yards will hang the said room?

From the content of the room, subtract the content of the windows, and divide the remainder by the square feet in a yard of tapestry, viz. $11\frac{1}{4}$ feet, and you will find the answer to be 83·59 yards.

4. If the axis of a globe be 27·5 inches, what are the solid and superficial contents?

Ans. $\begin{cases} 10889{\cdot}24375 \text{ inches} = 6{\cdot}3 \text{ feet solid.} \\ 2375{\cdot}835 \text{ inches} = 16{\cdot}49 \text{ feet superficies.} \end{cases}$

5. There is a segment of a sphere, the diameter of whose base is 24 inches, and its altitude 10 inches; what are the solid and superficial contents, including the area of its base?

Ans. $\begin{cases} 2785{\cdot}552 \text{ inches, the solidity.} \\ 1218{\cdot}9408 \text{ inches, the superficies.} \end{cases}$

6. If a tree girt 18 feet 6 inches, and be 24 feet long; how many tons of timber are contained in that tree, using the customary method of measuring, and allowing 40 feet of timber to a ton?

Ans. 12 tons 33 feet 4 inches 6 parts.

7. There is a cellar to be dug by the floor, the length of which is 33 feet 7 inches, and the breadth 18 feet 9 inches, and its depth is to be 5 feet 9 inches; how many floors of earth are in that cellar?

Ans. 11 floors 56 feet 8 inches 5 parts.

Note. A floor of earth is 324 solid feet.

8. There is a roof covered with tiles, whose depth on both sides (with the usual allowance at the eaves) is 35 feet 6 inches, and the length 48 feet 9 inches; how many squares of tiling are contained in it?

Ans. 17 squares 30 feet $7\frac{1}{2}$ inches.

9. There is a cone, the diameter at the base being 42 inches, and the perpendicular height 94 inches; it is required to cut off two solid feet from the top.

end of it; what length of the perpendicular must be cut off? *Ans.* 40·436 inches.

10. If a square piece of timber be 12 feet long, and the side of the square of the greater end 21 inches, the side of the square of the less end 3 inches; how far must I measure from the greater end, to cut off five solid feet? *Ans.* 1·872.

11. Three men bought a grind-stone of 40 inches diameter, which cost 20 shillings; of which sum the first man paid 9 shillings, the second 6 shillings, and the third 5 shillings; how much of the stone must each man grind down, proportionably to the money he paid?

Ans. The first man must grind down 5·167603 inches of the radius; the second, 4·832397 inches; and the third, the rest, which is 10 inches.

12. It is required to divide a cone into three equal parts, by sections parallel to the base, and to find the altitudes of the three parts, the height of the whole cone being 20 inches?

Ans. The upper part = 13·867 inches.
The middle part = 3·604 inches.
The lower part = 2·528 inches.

13. There is a cistern, whose depth is seven-tenths of the width, the length is 6 times the depth, and the solidity is 367·5 feet; what are the depth, width, and length, and how many bushels of corn will it hold?

Ans. The depth is 3·5 feet, width 5 feet, and length 21 feet; and it will hold 295 bushels 1 peck 4 pints of corn.

14. Suppose a bushel to be exactly round,
And the depth, when measured, 8 inches be found;
If the breadth 18 inches and half you discover,
This bushel is legal all *England* over*.

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* By Act of Parliament, made in 1697.

But a workman would make one of another frame;
Seven inches and a half the depth of the same;
Now say, of what length must the diameter be,
That it may with the former in measure agree?
Ans. 19·107 inches.

15. In the midst of a meadow well stored with grass,
I took just an acre to tether my ass;
How long must the cord be, that feeding all round,
He may'nt graze less nor more than his acre of ground?
Ans. 117 feet 9 inches.

16. A malster has a kiln, that is 16 feet 6 inches square; but he intends to pull it down, and build a new one, that may be large enough to dry three times as much at a time as the old one will do; what will be the side of the new kiln? *Ans.* 28·578 feet.

17. If a round cistern be 26·3 inches in diameter, and 52·5 inches deep; how many inches in diameter must a cistern be to hold twice the quantity, the depth being the same? And how many ale gallons will each cistern hold?

Ans. The diameter of the greater cistern is 37·19 inches, and its content 202·275 gallons; hence the content of the less cistern must be 101·137 gallons.

18. If the diameter of a cask at the bung be 32 inches, and at the head 25 inches, and the length 40 inches; how many ale gallons are contained therein?
Ans. 94·41 gallons by the general Rule, p. 233.

19. There is a stone 20 inches long, 15 inches broad, and 8 inches thick, which weighs 217 pounds; required the length, breadth, and thickness of another of the same kind and shape, which weighs 1000 pounds?

Ans. $\left\{\begin{array}{l}\text{The length 33·282 inches.}\\ \text{The breadth 24·961 inches.}\\ \text{The thickness 13·312 inches.}\end{array}\right.$

20. If an iron bullet, whose diameter is 4 inches, weighs 9 pounds; what will be the weight of another bullet (of the same metal) whose diameter is 9 inches? *Ans.* 102·515 pounds.

21. There is a square pyramid of marble, each side of its base is 5 inches, the height 15 inches, and its weight is 12 pounds and a quarter; required the weight of a similar pyramid, each side of whose base is 30 inches. *Ans.* The weight is 2646 pounds.

22. There is a ball or globe of marble, whose diameter is 6 inches, and its weight 11 pounds; what will be the diameter of another globe of the same kind of marble, that weighs 500 pounds?

Ans. 21·4 inches.

23. There is a frustum of a pyramid, whose bases are regular octagons; each side of the greater base is 21 inches, and each side of the less base is 9 inches, and its perpendicular length is 15 feet; how many solid feet are contained in it? *Ans.* 119·2 feet.

24. There is a frustum of a cone, the diameter of the greater base is 36 inches, the diameter of the less base 20 inches, and the length or height 215 inches; required the length and solid content of the whole cone, and also the solid content of the given frustum?

Ans. { The solidity of the whole cone = 94·98 feet.
The solidity of the frustum = 78·7 feet.

25. If the top part of a cone contains 26171 solid inches, and is 200 inches in length, and the lower frustum thereof contains 159610 solid inches; what is the length of the whole cone, and the diameter of each base?

Ans. { The length of the whole cone 384·3766 inches.
The diameter of the greater base 42·9671 in.
The diameter of the less base 22·3568 inches.

26. There is a frustum of a cone, whose solid content is 20 feet, and its length 12 feet; the greater

diameter is to the less as 5 to 2; what are the diameters?

Ans. { 2·02012 feet.
·80804 feet.

27. There is a room of wainscot 129 feet 6 inches in circumference, and 16 feet 9 inches high (being girt over the mouldings;) there are two windows, each 7 feet 3 inches high, and the breadth of each, from cheek to cheek, 5 feet 6 inches; the breadth of the shutters of each is 4 feet 6 inches; the cheek-boards and top and bottom boards of each window, taken together, 24 feet 6 inches, and their breadth 1 foot 9 inches; the door-case 7 feet high, and 3 feet 6 inches wide; the door 3 feet 3 inches wide; how many yards of wainscot are contained in that room?

	F.	I.	P.
The content of the room - - - - -	2169	1	6
The shutters, at work and half - - -	97	10	6
The door, at work and half - - - - -	34	1	6
The cheek-boards - - - - - - - - -	85	9	0
The sum	2386	10	6
The window-lights and door-case deduct	104	3	0
	9)2282	7	6
	253	5	0

Ans. 253 yards 5 feet.

28. There is a wall which contains 18225 cubic feet, the height is 5 times the breadth, and the length 8 times the height; what are the length, breadth, and height?

Ans. { The length = 180 feet.
The breadth = 4·5 feet.
The height = 22·5 feet.

29. The upper end of a May-pole was broken off by a blast of wind, and, in falling, struck the ground at 15 feet distance from the foot of the part standing, the broken piece was 39 feet in length, what was the length of the May-pole? *Ans.* 75 feet.

30. A May-pole there was, whose height I would know,
The sun shining clear, straight to work I did go:
The length of the shadow, upon level ground,
Just sixty-five feet, when measured, I found;
A staff I had there, just five feet in length;
The length of its shadow was four feet one-tenth:
How high was the May-pole, I gladly would know?
And it is the thing you're desired to show.
Ans. 79·26 feet.

31. What will be the diameter of a globe, when the solidity and superficial content of it are equal?
Ans. 6: when the diameter of the globe is 1, the solidity is to the superficies as 1 to 6.

32. What will the axis of a globe be, when the solidity is to the superficies, as 9 to 4? *Ans.* 13·5.

33. There are three grenado-shells, of such capacity, that the second shell will just lie in the concavity of the first, and the third in the concavity of the second. The solidity of the metal of the first shell is equal to the number of solid inches which would fill its concavity: the solidity of the metal of the second shell is in proportion to the number of solid inches which would fill its concavity as 7 is to 5; and the solidity of the metal of the third shell is in proportion to the number of solid inches which would fill its concavity as 9 is to 4. The diameter of the first shell is 16 inches; required the diameters of the second and third shells, the thickness of each shell, and also the weight, supposing a solid inch of iron to weigh 4 ounces avoirdupois?

Answer.

$$\text{The diameter of the}\left\{\begin{array}{l}\text{First shell} = 16\cdot\ \text{inches.}\\ \text{Second shell} = 12\cdot699\ \text{inches,}\\ \text{Third shell} = 9\cdot485\ \text{inches.}\end{array}\right.$$

$$\begin{array}{c}\text{The thickness of the}\\ \text{metal of the}\end{array}\left\{\begin{array}{l}\text{First shell} = 1\cdot65\ \text{inches.}\\ \text{Second shell} = 1\cdot607\ \text{inches,}\\ \text{Third shell} = 1\cdot541\ \text{inches.}\end{array}\right.$$

$$\text{The weight of the}\left\{\begin{array}{l}\text{First shell} = 268\cdot08\ \text{pounds.}\\ \text{Second shell} = 156\cdot38\ \text{pounds.}\\ \text{Third shell} = 77\cdot33\ \text{pounds.}\end{array}\right.$$

The internal diameter of the first shell is equal to the external diameter of the second; and the internal diameter of the second, is equal to the external diameter of the third.

34. On Shrove-Tuesday last, I'll tell you what pass'd
In a neighbouring gentleman's kitchen,
Where pancakes were making, with eggs and with bacon,
As good as e'er cut off a *flitchen.*
The cook-maid she makes four lusty pancakes,
For *William*, her fav'rite gard'ner;
" *Pray* be quick with your four," cries Jack, "and make more,
For William won't let me go partner."
Being sparing of lard, the pan's bottom she marr'd,
In making the last of Will's four,
So she said, " *Pry'thee*, John, run and borrow a pan,
Or else I can't make any more."
Jack soon got a pan, but he found by his span,
That the first was more wide than the latter;
This being a foot o'er, whereas that before
Was three inches more, and a quarter.
Jack cries, " *Don't* me cozen, but make half a dozen,
For the pan is much less than before:"
Says Will, " For a *crown*, and I'll put the cash down,
Your *six* will be more than my *four*."

" 'Tis done," says brisk Jack, and his *crown* he did
stake,
So both of them sent for a *gauger:*
The dimensions he takes of all their pancakes,
To determine this *important* wager:
He found by his stick, they were *equally thick*,
So one of Will's cakes he did take,
Which he straight cut in twain, twelve-one-fifth the
chord-line *,
And gave the *less* piece unto Jack:
" To the best of *my* skill," says the gauger "this will
Make both your shares equal and true."
Will swore that he ly'd; so the point to decide,
They refer themselves, *Sirs*, unto you.
Then pray give your answers, as soon as you can, *Sirs*,
For what with their quarrels and jars,
We're afraid of some murder, for no day goes over,
But they fight, and are cover'd with scars.

New London Magazine, 1786, p. 158.

Ans. The versed sine of the segment of *Will's* cake, which was given to *Jack*, was 3·05 inches, and its area 26·0058 square inches; hence, *Will's* share was 704·6125 square inches; and *Jack's* share 704·5914 square inches; so that *Will's* four were about 26 square inches more than *Jack's* six; and *Will* of course lost the wager. After the decision of the gauger, *Will's* share was nearly $\frac{1}{30}$ of a square inch more than *Jack's*.

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* $12\frac{1}{5}$ inches.

A
TABLE
OF THE
AREAS
OF THE
SEGMENTS OF A CIRCLE,

Whose diameter is unity, and supposed to be divided into 1000 *equal parts.*

Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.
·001	·000042	·025	·005230	·049	·014247
·002	·000119	·026	·005546	·050	·014681
·003	·000219	·027	·005867	·051	·015119
·004	·000337	·028	·006194	·052	·015561
·005	·000470	·029	·006527	·053	·016007
·006	·000618	·030	·006865	·054	·016457
·007	·000779	·031	·007209	·055	·016911
·008	·000951	·032	·007558	·056	·017369
·009	·001135	·033	·007913	·057	·017831
·010	·001329	·034	·008273	·058	·018296
·011	·001533	·035	·008638	·059	·018766
·012	·001746	·036	·009008	·060	·019239
·013	·001968	·037	·009383	·061	·019716
·014	·002199	·038	·009763	·062	·020196
·015	002438	·039	·010148	·063	·020691
·016	·002685	·040	·010537	·064	·021168
·017	·002940	·041	·010931	·065	·021659
·018	·003202	·042	·011330	·066	·022154
·019	·003471	·043	·011734	·067	·022652
·020	·003748	·044	·012142	·068	·023154
·021	·004031	·045	·012554	·069	·023659
·022	·004322	·046	·012971	·070	·024168
·023	·004618	·047	·013392	·071	·024680
·024	·004921	·048	·013818	·072	·025195

Height.	Area Seg	Height	Area Seg.	Height.	Area Seg.
·073	·025714	·111	·047632	·149	·073161
·074	·026236	·112	·048262	·150	·073874
·075	·026761	·113	·048894	·151	·074589
·076	·027289	·114	·049528	·152	·075306
·077	·027821	·115	·050165	·153	·076026
078	·028356	·116	·050804	·154	·076747
·079	·028894	·117	·051446	·155	·077469
·080	·029435	·118	·052090	·156	·078194
·081	·029979	·119	·052736	·157	·078921
·082	·030526	·120	·053385	·158	·079649
·083	·031076	·121	·054036	·159	·080380
·084	·031629	·122	·054689	·160	·081112
·085	·032180	·123	·055345	·161	·081846
·086	·032745	·124	·056003	·162	·082582
·087	·033307	·125	·056663	·163	·083320
·088	·033872	·126	·057326	·164	·084059
·089	·034441	·127	·057991	·165	·084801
·090	·035011	·128	·058658	·166	·085544
·091	·035585	·129	·059327	·167	·086289
·092	·036162	·130	·059999	·168	·087036
·093	·086741	·131	·060672	·169	·087785
·094	·037323	·132	·061348	·170	·088535
·095	·037909	·133	·062026	·171	·089287
·096	·038496	·134	·062707	·172	·090041
·097	·039087	·135	·063389	·173	·090797
·098	·039580	·136	·064074	·174	·091554
·099	·040276	·137	·064760	·175	·092313
·100	·040875	·138	·065449	·176	·093074
·101	·041476	·139	·066140	·177	·093836
·102	·042080	·140	·066833	·178	·094601
·103	·042687	·141	·067528	·179	·095366
·104	·043296	·142	·068225	·180	·096134
·105	·043908	·143	·068924	·181	·096903
·106	·044522	·144	·069625	·182	·097674
·107	·045139	·145	·070328	·183	·098447
·108	·045759	·146	·071033	·184	·099221
·109	·046381	·147	·071741	·185	·099997
·110	·047005	·148	·072450	·186	·100774

Height.	Area Seg.	Height.	Area Seg.	Height.	Area Seg.
·187	·101553	·225	·132272	·263	·164899
·188	·102334	·226	·133108	·264	·165780
·189	·103116	·227	·133945	·265	·166663
·190	·103900	·228	·134784	·266	·167546
·191	·104685	·229	·135624	·267	·168430
·192	·105472	·230	·136465	·268	·169315
·193	·106261	·231	·137307	·269	·170202
·194	·107051	·232	·138150	·270	·171089
·195	·107842	·233	·138995	·271	·171978
·196	·108636	·234	·139841	·272	·172867
·197	·109430	·235	·140688	·273	·173758
·198	·110226	·236	·141537	·274	·174649
·199	·111024	·237	·142387	·275	·175542
·200	·111823	·238	·143238	·276	·176435
·201	·112624	·239	·144091	·277	·177330
·202	·113426	·240	·144944	·278	·178225
·203	·114230	·241	·145799	·279	·179122
·204	·115035	·242	·146655	·280	·180019
·205	·115842	·243	·147512	·281	·180918
·206	·116650	·244	·148371	·282	·181817
·207	·117460	·245	·149230	·283	·182718
·208	·118271	·246	·150091	·284	·183619
·209	·119083	·247	·150953	·285	·184521
·210	·119897	·248	·151816	·286	·185425
·211	·120712	·249	·152680	·287	·186329
·212	·121529	·250	·153546	·288	·187234
·213	·122347	·251	·154412	·289	·188140
·214	·123167	·252	·155280	·290	·189047
·215	·123988	·253	·156149	·291	·189955
·216	·124810	·254	·157019	·292	·190864
·217	·125634	·255	·157890	·293	·191775
·218	·126459	·256	·158762	·294	·192684
·219	·127285	·257	·159636	·295	·193596
·220	·128113	·258	·160510	·296	·194509
·221	·128942	·259	·161386	·297	·195422
·222	·129773	·260	·162263	·298	·196337
·223	·130605	·261	·163140	·299	·197252
·224	·131438	·262	·164019	·300	·198168

Height.	Area Seg.	Height.	Area Seg	Height.	Area Seg.
·301	·199085	·339	·234526	·377	·270951
·302	·200003	·340	·235473	·378	·271920
·303	·200922	·341	·236421	·379	·272890
·304	·201841	·342	·237369	·380	·273861
·305	·202761	·343	·238318	·381	·274832
·306	·203683	·344	·239268	·382	·275803
·307	·204605	·345	·240218	·383	·276775
·308	·205527	·346	·241169	·384	·277748
·309	·206451	·347	·242121	·385	·278721
·310	·207376	·348	·243074	·386	·279694
·311	·208301	·349	·244026	·387	·280668
·312	·209227	·350	·244980	·388	·281642
·313	·210154	·351	·245934	·389	·282617
·314	·211082	·352	·246889	·390	·283592
·315	·212011	·353	·247845	·391	·284568
·316	·212940	·354	·248801	·392	·285544
·317	·213871	·355	·249757	·393	.286521
·318	·214802	·356	·250715	·394	·287498
·319	·215733	·357	·251673	·395	·288476
·320	·216666	·358	·252631	·396	·289453
·321	·217599	·359	·253590	·397	·290432
·322	·218533	·360	·254550	·398	·291411
·323	·219468	·361	·255510	·399	·292390
·324	·220404	·362	·256471	·400	·293369
·325	·221340	·363	·257433	·401	·294349
·326	·222277	·364	·258395	·402	·295330
·327	·223215	·365	·259357	·403	·296311
·328	·224154	·366	·260320	·404	·297292
·329	·225093	·367	·261284	·405	·298273
·330	·226033	·368	·262248	·406	·299255
·331	·226974	·369	·263213	·407	·300238
·332	·227915	·370	·264178	·408	·301220
·333	·228858	·371	·265144	·409	·302203
·334	·229801	·372	·266111	·410	·303187
·335	·230745	·373	·267078	·411	·304171
·336	·231689	·374	·268045	·412	·305155
·337	·232634	·375	·269013	·413	·306140
·338	·233580	·376	·269982	·414	·307125

Height.	Area Seg.	Height.	Area Seg.	Height	Area Seg.
·415	·308110	·444	·336816	·473	·365712
·416	·309095	·445	·337810	·474	·366710
·417	·310081	·446	·338804	·475	·367709
·418	·311068	·447	·339798	·476	·368708
·419	·312054	·448	·340793	·477	·369707
·420	·313041	·449	·341787	·478	·370706
·421	·314029	·450	·342782	·479	·371705
·422	·315016	·451	·343777	·480	·372704
·423	·316004	·452	·344772	·481	·373703
·424	·316992	·453	·345768	·482	·374702
·425	·317981	·454	·346764	·483	·375702
·426	·318970	·455	·347759	·484	·376702
·427	·319959	456	·348755	·485	·377701
·428	·320948	·457	·349752	·486	·378701
·429	·321938	·458	·350748	·487	·379700
·430	·322928	·459	·351745	·488	·380700
·431	·323918	·460	·352742	·489	·381699
·432	·324909	·461	·353739	·490	·382699
·433	·325900	·462	·354736	·491	·383699
·434	·326892	·463	·355732	·492	·384699
·435	·327882	·464	·356730	·493	·385699
·436	·328874	·465	·357727	·494	·386699
·437	·329866	·466	·358726	·495	·387699
·438	·330858	·467	·359723	·496	·388699
·439	·331850	·468	·360721	·497	·389699
·440	·332843	·469	·361719	·498	·390699
·441	·333836	·470	·362717	·499	·391699
·442	·334829	·471	·363715	·500	·392699
·443	·335822	·472	·364713		

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